The Complexity of Succinct Elections

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Abstract

The computational study of elections generally assumes that the preferences of the electorate come in as a list of votes. Depending on the context, it may be much more natural to represent the preferences of the electorate succinctly, as the distinct votes and their counts. Though the succinct representation may be exponentially smaller than the nonsuccinct, we find only one natural case where the complexity increases, in sharp contrast to the case where each voter has a weight, where the complexity usually increases.

1 Introduction

Elections are an important and widely used tool for determining an outcome given the preferences of several agents. It is well known that every reasonable election system is manipulable, however, it may be computationally infeasible to determine if such a manipulation exists. Bartholdi, Tovey, and Trick (1989) started the computational study of manipulation problem and later introduced the study of a family of manipulative attacks, denoted control, which models the actions of an election chair with control over the structure of the election who wants to ensure his or her preferred outcome (Bartholdi, Tovey, and Trick 1992).

In most of the computational studies on elections, the preferences of the voters are represented as a list of votes. Though this may be a reasonable representation for paper ballots in political elections, in artificial intelligence applications a more succinct representation where the preferences of the electorate are represented as a list of distinct votes and their counts may be more natural. For example, this representation is used by the online preference repository PrefLib for election data (Mattei and Walsh 2013).

We consider how this succinct representation of the votes can affect the complexity of different election problems, and contrast this with the case of weighted voters.

Though the succinct representation may be exponentially smaller than the nonsuccinct representation, we find that in surprisingly few cases the complexity increases. Related work that considers succinct votes did not find a case where the complexity increases (Faliszewski et al. 2009; Faliszewski, Hemaspaandra, and Hemaspaandra 2009; Hemaspaandra, Hemaspaandra, and Rothe 2009; Faliszewski et al. 2011). We explain this phenomenon by showing that many common proof techniques that show that election problems are in P can be adapted to the succinct case. We found only one natural case where the complexity increases: The complexity of determining the winner in a Kemeny election.

For details, see the full version (Fitzsimmons and Hemaspaandra 2016).

2 Preliminaries

A *(nonsuccinct) election* is defined as a set of candidates C, and a list of voters V, where each voter $v \in V$ has a preference order over the candidates. In a *succinct election* V is not a list of voters, but instead a list of distinct votes v (preference orders) and their positive integer count $\kappa(v)$. In a *weighted election*, V is a list of voters and each $v \in V$ has a positive integer weight $\omega(v)$ and can be thought of as a coalition of $\omega(v)$ voters all voting the same. We assume each voter has total order preferences, i.e., he or she strictly ranks each candidate from most to least preferred.

An election system $\mathcal{E}$ maps an election to set of winners, where the winners can be any subset of the candidate set. The problem $\mathcal{E}$-Winner is defined in the following way. Given an election (C, V) and a candidate $p \in C$, is p a winner of (C, V) using election system $\mathcal{E}$?

For $\mathcal{E}$-Succinct-Winner, V is represented succinctly, and for $\mathcal{E}$-Weighted-Winner, the election is weighted.

A scoring vector $\langle \alpha_1, \alpha_2, \ldots, \alpha_m \rangle$, $\alpha_i \geq \alpha_{i+1}$ defines an election system over m candidates. Each candidate receives α_i points for each vote where they are ranked ith, and the candidate(s) with the highest score win.

A *pure scoring rule* defines a family of scoring vectors where the m-candidate scoring vector can be computed in polynomial time in m, and the $m+1$-candidate scoring vector can be obtained from the m-candidate scoring vector by adding a single coefficient (Betzler and Dorn 2009).

Control denotes the family of manipulative actions that consider an agent with control over the structure of the election who wants to ensure his or her preferred outcome (Bartholdi, Tovey, and Trick 1992). Constructive control by adding voters (CCAV) is a very natural case of control and can be thought of as modeling get-out-the-vote drives, and is defined as follows: Given a set of candidates C, a list of registered voters V, a list of unregistered voters U, an add limit k, and a preferred candidate $p \in C$, does there exist a list U' of unregistered voters $U' \subseteq U$ such that U' consists of at most k unregistered voters and p is a winner of $(C, V \cup U')$ using election system $\mathcal{E}$?

In the standard model of weighted voter control the parameter k denotes the number of weighted voters the chair can add/delete (Faliszewski, Hemaspaandra, and Hemaspaandra 2015). In the succinct case, the only change from the CCAV definition above is that the voters (registered and unregistered) are represented succinctly.

3 Adapting Approaches

The general theme of this paper is that in surprisingly few cases, the complexity of election problems increases when we allow succinct representation. Several common approaches used to show that election problems are in P (for the nonsuccinct case) can be adapted for the case where voters are represented succinctly, sometimes straightforwardly and sometimes in a more complicated way.

In the full version of this paper, we show how greedy approaches, limited brute-forcing, network flow, and edge matching/cover techniques can be adapted (Fitzsimmons and Hemaspaandra 2016). To showcase these adaptations, we show that the dichotomy result for CCAV for pure scoring rules (Hemaspaandra, Hemaspaandra, and Schnoor 2014) holds for the succinct case. Assuming P $\neq$ NP, the following pure scoring rules are asymptotically the only cases where CCAV is in P, both for the nonsuccinct and the succinct case.

- $\langle \alpha, \beta, 0, \ldots, 0 \rangle$, where $\alpha > \beta$.
- t-Approval, $\langle 1^t, 0, \ldots, 0 \rangle$, where $t \leq 3$.
- t-Veto, $\langle 1, \ldots, 1, 0^t \rangle$, where $t \leq 2$.
- $\langle 2, 1, \ldots, 1, 0 \rangle$.

In contrast, all weighted cases are NPC, except triviality ($\langle 0, \ldots, 0 \rangle$), 1-approval, 2-approval, and 1-veto (Faliszewski, Hemaspaandra, and Hemaspaandra 2015; Lin 2012).

Weighted voter problems are usually hard, even when the number of candidates is fixed. In contrast, Faliszewski et al. (2009) showed that for succinct votes, manipulation is in P for any fixed number of candidates by describing an integer linear program with a fixed number of variables. This approach can also be used to show succinct constructive control by adding/deleting voters is in P for every scoring rule for any fixed number of candidates.

4 Kemeny Elections

Kemeny elections were introduced in (Kemeny 1959). A candidate p is a Kemeny winner if p is ranked first in a Kemeny consensus. A Kemeny consensus is a linear order $>$ over the candidates that minimizes the Kendall's tau distance to V, i.e., that minimizes $\sum_{a,b \in C, a > b} \|\{v \in V \mid b >_v a\}\|$, where $>_v$ is the preference order of voter v. Observe that Kemeny-Weighted-Winner is equivalent to Kemeny-Succinct-Winner.

Hemaspaandra et al. (2005) showed that Kemeny-Winner is Θ_2^p-complete and Footnote 3 in their paper points out that Kemeny-Succinct-Winner is in Δ_2^p and explicitly leaves the exact complexity of this problem as an open question.[1]

We show that Kemeny-Succinct-Winner (and Kemeny-Weighted-Winner) are in fact Δ_2^p-complete by "lifting" the chain of reductions used by Hemaspaandra et al. (2005) between Θ_2^p-complete problems. We accomplish this by defining Δ_2^p-complete weighted versions of the two intermediate problems and show that the weighted versions of the reductions still hold. This works surprisingly well. However, showing that the initial problem in our chain is Δ_2^p-complete is much more involved.

5 Conclusions and Future Work

Overall, we found that when we allow succinct representation of the voters that the complexity of election problems rarely increases, since several common techniques for showing election problems in P generalize. The only cases where we found an increase when allowing a succinct representation had equivalent weighted and succinct problems. An interesting open question is if this will always be the case.

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[1] Δ_2^p denotes the class of problems solvable in P with access to an NP oracle. Θ_2^p is a subset of Δ_2^p, where the P-machine can ask one round of parallel queries to its NP oracle.