Insight into Voting Problem Complexity Using Randomized Classes

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Motivation

- An important direction in the computational study of elections is studying the complexity of voting problems such as electoral control.
- The first step in classifying a voting problem is showing the problem easy or hard.
- Easy usually means P, but there are alternative definitions such as RP (randomized polynomial time).
- We show that RP may be the right notion of easy for several natural election problems.

RP and Matching

RP is the class of languages for which there exists a probabilistic p-time algorithm that can have false negatives, but not false positives.

Widely assumed that $\mathrm{P} = \mathrm{RP}$, but has not been shown.

Our results link the complexity of voting problems to the following well-studied matching problem.

Exact Perfect [Bipartite] Matching [PY82]:

> **Given:** A [bipartite] graph $G = (V, E)$, a set $R \subseteq E$ of red edges, and an integer $k \geq 0$.
>
> **Question:** Does there exist a perfect matching of G with exactly k red edges?

In RP by [MVV87]. Has not been shown to be in P.

Electoral Control

We consider several types of electoral control, which ask if an agent can modify the structure of an election to ensure a preferred outcome.

In Control by Adding Voters (CCAV) [BTT92] we ask if there is a subset of at most k unregistered voters to add to an election to ensure a preferred candidate wins.

In Control by Replacing Voters (CCRV) [LNRVW15] we ask if there is a subset of at most k voters to replace in an election with the same number of unregistered voters to ensure a preferred candidate wins.

We also consider the *exact* variants of each of these problems.

Results

	First-Last $\langle 1, 0, \ldots, 0, -1 \rangle$
CCAV	P [HHS14]
CCAV!	equiv. to Exact Perfect Bipartite Matching
CCRV	equiv. to Exact Perfect Bipartite Matching
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	2-Approval $\langle 1, 1, 0, \ldots, 0 \rangle$
CCAV	P [Lin11]
CCAV!	P
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Our results establish even better upper bounds than RP, since Exact Perfect Bipartite Matching and Exact Perfect Matching are not only in RP, but even in RNC [MVV87].

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Thank you!