College of the Holy Cross, Fall Semester, 2016
Math 242, Midterm 2 Practice Questions

1. Give an example of each of the following, or explain why such an example is not possible.
   (a) An unbounded Cauchy sequence.
   (b) A bounded sequence that has no convergent subsequences.
   (c) A convergent sequence that has a subsequence that converges to 5 and a subsequence that converges to 7.
   (d) Series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ that both diverge, such that the series $\sum_{n=1}^{\infty} a_n + b_n$ converges.
   (e) A series $\sum_{n=1}^{\infty} a_n$ that converges and a series $\sum_{n=1}^{\infty} b_n$ that diverges, such that the series $\sum_{n=1}^{\infty} a_n + b_n$ converges.
   (f) An unbounded sequence x_n with $x_n \geq 0$ for all n such that $\lim x_n \neq +\infty$.
2. Use the definition of an infinite limit to prove that $\lim \sqrt{1+\sqrt{n}} = +\infty$.
3. Suppose $\lim x_n = +\infty$ and $\lim \frac{x_n}{y_n} = 3$. Prove that $\lim y_n = +\infty$.
4. Find the sum of each series. Explain your reasoning.
   (a) $\displaystyle\sum_{n=0}^{\infty} \frac{9^{n-1} + 10^{n+1}}{11^n}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \cos(\pi/n) - \cos(\pi/(n+2))$
5. Suppose a_n and b_n are nonnegative for all n, $\lim \frac{a_n}{b_n} = \frac{3}{4}$ and $\displaystyle\sum_{n=1}^{\infty} b_n = 8$. Which of the following statements are true?
   (i) $\displaystyle\sum_{n=1}^{\infty} a_n = 6$.
   (ii) $\displaystyle\sum_{n=1}^{\infty} a_n$ converges.
   (iii) $\displaystyle\sum_{n=1}^{\infty} a_n$ diverges.
   (iv) This doesn't tell us anything about $\displaystyle\sum_{n=1}^{\infty} a_n$.

6. State whether each of the following series converges or diverges. Prove your assertion.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{4+n}{1+n^3}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{2}{5n+3}$

(d) $\displaystyle\sum_{n=1}^{\infty} \cos\left(\frac{\pi}{n}\right)$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{(\ln n)^2}{n^2}$

7. Suppose $a_n > 0$ for all n and $\displaystyle\sum_{n=1}^{\infty} a_n$ converges. Prove that $\displaystyle\sum_{n=1}^{\infty} a_n^2$ converges.

8. Find the interval of convergence of each power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(x+2)^n}{\sqrt{n}}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{2^n+3^n}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(n!)^2 x^n}{(2n)!}$

9. Suppose that $\displaystyle\lim_{n\to\infty} a_n = \frac{3}{5}$. Prove that the interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} a_n x^n$ is $(-1, 1)$.