Syllabus for MATH 410, Field and Galois Theory

College of the Holy Cross, Spring 2025

Instructor: Dr. Neranga Fernando

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Office Hours: Anytime I am in the office

Meeting Times and Location: TBD (2 hours and 30 minutes a week), TBD

Course Description: The course begins with the Fundamental Theorem of Fields. Fields and field extensions are discussed in detail. The Fundamental Theorem of Galois Theory is followed by a discussion of some Galois extensions and applications of Galois Theory.

Course Objectives: To learn the fundamentals of fields, field extensions, the Galois group of a polynomial, and solvability of polynomials by radicals.

Prerequisites: Modern Algebra (MATH 351) and Linear Algebra (MATH 244)

Recommended Reading: Field and Galois Theory by Patrick Morandi, Graduate Texts in Mathematics (Springer)

Course Materials: All announcements, materials and grades will be posted on Canvas.

Homework: There will be five homework assignments during the semester.

By doing mathematics you learn mathematics. You learn math best when you approach the subject as something you enjoy. Learn to explain mathematics to your classmates. Mathematics can be fun and rewarding when there are people around you who enjoy figuring out problems as much as you do. Take advantage of this opportunity and organize study groups. I will not consider working on homework problems with your classmates as a violation of the academic honesty policy in the department. However, you must prepare and submit your own solutions.
Please follow these guidelines when you submit homework assignments:

- Put your name, the date, and the homework assignment number at the top of the first page.
- Staple multi-page assignments.
- Write neatly and show all your work.
- On the last page of your assignment, please write the name(s) of your classmate(s) with whom you work on homework problems (with an asterisk).
- Make sure you attach the honor code.

End-of-semester Paper: On the last day of classes (May 5, 2025), the student is required to submit a 10-page paper (double-spaced) on the research project: Polynomials defined by functional equations over finite rings.

End-of-semester Presentation: The student is required to give a talk to the department and math majors on the Academic Conference Day (April 23, 2025) about his/her research project and results.

Grading: The course grade will be determined as follows:
Homework: 70% (14% each)
End-of-semester Presentation: 15%
End-of-semester Paper: 15%

Academic Honesty: A necessary prerequisite to the attainment of the goals of the College is maintaining complete honesty in all academic work. Students are expected to present their own work in exams and in any material submitted for credit. Students may not assist others in presenting work that is not their own. Offenders are subject to disciplinary action. A violation of the Department Policy on Academic Integrity will result in a 0 for that quiz or exam, and a letter describing the occurrence of academic dishonesty will be sent to the Chair of the Department of Mathematics and Computer Science and your Class Dean.

For more on Academic Integrity see:

`https://www.holycross.edu/academics/programs/mathematics-and-computer-science/node/211581/academic-integrity`

Note: The student is expected to read a few chapters on rings in Contemporary Abstract Algebra (ninth edition) by Joseph A. Gallian during the winter break. The chapters will be assigned to the student before the winter break.

Important Dates:

March 3 – 7	Spring Break: no classes
April 17, 18 & 21	Easter Break: no classes
April 23	Academic Conference Day: no classes
May 5	Last day of classes

The mind is not a vessel to be filled but a fire to be kindled.

— Plutarch

Schedule of Topics

- A quick review of rings
- The Fundamental Theorem of Field Theory
- Splitting Fields
- Zeros of an Irreducible Polynomial
- Field Extensions
- Automorphisms
- Normal Extensions
- Separable and Inseparable Extensions
- The Fundamental Theorem of Galois Theory
- Finite Fields
- Cyclotomic Extensions
- Norms and Traces
- Cyclic Extensions
- Hilbert Theorem 90
- Kummer Extensions
- Discriminants
- Polynomials of Degree 3 and 4
- The Transcendence of π and e
- Ruler and Compass Constructions
- Solvability of Polynomials by Radicals
- Insolvability of a Quintic
- **Research Project:** Polynomials defined by functional equations over finite rings