

Syllabus for MATH 400, Representation Theory

College of the Holy Cross, Spring 2024

Instructor: Dr. Neranga Fernando

Office: Haberlin 310

E-mail: nfernand@holycross.edu

Office Phone: 508-793-2458

Office Hours: Anytime I am in the office

Meeting Times and Location: TBD (2 hours and 30 minutes a week), TBD

Course Description: Representation theory is concerned with the ways of writing a group as a group of matrices. It provides a key to a proper understanding of finite groups.

Course Objectives: To learn the fundamentals of representations and characters of finite groups.

Prerequisites: Modern Algebra (MATH 351) and Linear Algebra (MATH 244)

Recommended Reading: Representations and Characters of Groups by Gordon James and Martin Liebeck (second edition). It is absolutely NOT required that you purchase a hard copy of the text.

Course Materials: All announcements, materials and grades will be posted on Canvas.

Homework: There will be five homework assignments during the semester.

By doing mathematics you learn mathematics. You learn math best when you approach the subject as something you enjoy. Learn to explain mathematics to your classmates. Mathematics can be fun and rewarding when there are people around you who enjoy figuring out problems as much as you do. Take advantage of this opportunity and organize study groups. I will not consider working on homework problems with your classmates as a violation of the academic honesty policy in the department. However, you must prepare and submit your own solutions.

Please follow these guidelines when you submit homework assignments:

- Put your name, the date, and the homework assignment number at the top of the first page.
- Staple multi-page assignments.
- Write neatly and show all your work.
- On the last page of your assignment, please write the name(s) of your classmate(s) with whom you work on homework problems (with an asterisk).
- Make sure you attach the honor code.

Grading: The course grade will be determined as follows:

Homework: 100% (20% each)

Academic Honesty: A necessary prerequisite to the attainment of the goals of the College is maintaining complete honesty in all academic work. Students are expected to present their own work in exams and in any material submitted for credit. Students may not assist others in presenting work that is not their own. Offenders are subject to disciplinary action. A violation of the Department Policy on Academic Integrity will result in a 0 for that quiz or exam, and a letter describing the occurrence of academic dishonesty will be sent to the Chair of the Department of Mathematics and Computer Science and your Class Dean.

For more on Academic Integrity see:

<https://www.holycross.edu/academics/programs/mathematics-and-computer-science/node/211581/academic-integrity>

Important Dates:

March 4 – 8	Spring Break: no classes
March 28, 29 and April 1	Easter break: no classes
May 6	Last day of classes

The mind is not a vessel to be filled but a fire to be kindled.

— Plutarch

Schedule of Topics

- A quick review of groups and homomorphisms
- Group Representations
- FG -modules
- FG -submodules and reducibility
- Group Algebras
- FG -Homomorphisms
- Maschke's Theorem
- Schur's Lemma
- Irreducible modules and the group algebra
- Conjugacy classes
- Characters
- Inner products of characters
- The number of irreducible characters
- Character tables and orthogonality relations
- Normal subgroups and lifted characters
- Tensor products
- Restriction to a subgroup
- Characters of groups of order pq
- Characters of some p -groups