

1. Find the derivative of the function using the definition of derivative. State the domain of the function and the domain of its derivative.

$$G(t) = \frac{1 - 2t}{3 + t}$$

2. Calculate y' .

$$(i) y = (x^2 + x^3)^4$$

$$(ii) y = \frac{x^2 - x + 2}{\sqrt{x}}$$

$$(iii) y = x^2 \sin(\pi x)$$

$$(iv) y = \ln(x \ln x)$$

$$(v) y = \frac{e^{1/x}}{x^2}$$

$$(vi) y = \sqrt{\arctan x}$$

$$(vii) y = \tan\left(\frac{t}{1 + t^2}\right)$$

$$(viii) y = 3^{x \ln x}$$

$$(ix) y = e^{x \sec x}$$

$$(x) y = \log_5(1 + 2x)$$

$$(xi) y = \ln(\sin x) - \frac{1}{2} \sin^2 x$$

$$(xii) y = x \tan^{-1}(4x)$$

$$(xiii) y = \cot(3x^2 + 5)$$

$$(xiv) y = \frac{\sqrt{x+1}(2-x)^5}{(x+3)^7}$$

$$(xv) y = \sin\left(\tan \sqrt{1+x^3}\right)$$

$$(xvi) y = (1 - x^{-1})^{-1}$$

$$(xvii) y = \tan^2(\sin \theta)$$

$$(xviii) y = \frac{1}{\sqrt{x}} - \frac{1}{\sqrt[5]{x^3}}$$

$$(xix) y = \frac{\tan x}{1 + \cos x}$$

$$(xx) y = x \cos^{-1} x$$

$$(xxi) y = \ln \sec x$$

$$(xxii) y = \cot(\csc x)$$

$$(xxiii) y = e^{x \tan x}$$

$$(xxiv) y = \frac{x}{2 - \tan x}$$

$$(xxv) y = \left(x + \frac{1}{x}\right)^5$$

$$(xxvi) f(t) = 2^{t^3}$$

$$(xxvii) s(t) = \sqrt{\frac{1 + \sin t}{1 + \cos t}}$$

$$(xxviii) y = \cot^2(\sin \theta)$$

$$(xxix) y = \cos\left(\frac{1 - e^{2x}}{1 + e^{2x}}\right)$$

$$(xxx) y = \sqrt{x + \sqrt{x + \sqrt{x}}}$$

$$(xxxi) y = 2^{3^{4^x}}$$

$$(xxxii) f(t) = \sin^2(e^{\sin^2 t})$$

$$(xxxiii) y = \sqrt{1 + xe^{-2x}}$$

$$(xxxiv) y = \cos \sqrt{\sin(\tan \pi x)}$$

$$(xxxv) y = [x + (x + \sin^2 x)^3]^4$$

$$(xxxvi) f(t) = \tan(\sec(\cos t))$$

3. Find equations of the tangent line and normal line to the given curve at the specified point.

$$(a) y = 2xe^x, (0, 0)$$

$$(b) y = \frac{2x}{x^2 + 1}, (1, 1)$$

4. If $g(x) = x/e^x$, find $g^{(n)}(x)$.

5. How many tangent lines to the curve $y = x/(x + 1)$ pass through the point $(1, 2)$? At which points do these tangent lines touch the curve?

6. Find equations of the tangent lines to the curve

$$y = \frac{x - 1}{x + 1}$$

that are parallel to the line $x - 2y = 2$.

7. Find $R'(0)$, where

$$R(x) = \frac{x - 3x^3 + 5x^5}{1 + 3x^3 + 6x^6 + 9x^9}$$

Hint: Instead of finding $R'(x)$ first, let $f(x)$ be the numerator and $g(x)$ the denominator of $R(x)$ and compute $R'(0)$ from $f(0)$, $f'(0)$, $g(0)$, and $g'(0)$.

8. Find $Q'(0)$, where

$$Q(x) = \frac{1 + x + x^2 + xe^x}{1 - x + x^2 - xe^x}$$

9. Prove that $\frac{d}{dx}(\tan x) = \sec^2 x$

11. Prove that $\frac{d}{dx}(\sec x) = \sec x \tan x$

10. Prove that $\frac{d}{dx}(\cot x) = -\csc^2 x$

12. Prove that $\frac{d}{dx}(\csc x) = -\csc x \cot x$

13. For what values of x does the graph of f have a horizontal tangent?

(a) $f(x) = x + 2 \sin x$

(b) $f(x) = e^x \cos x$

14. Find an equation of the tangent line to the curve at the given point.

(a) $y = \sin x + \cos x$, $(0, 1)$

(c) $y = x + \tan x$, (π, π)

(e) $y = xe^{-x^2}$, $(0, 1)$

(b) $y = e^x \cos x$, $(0, 1)$

(d) $y = \sin(\sin x)$, $(\pi, 0)$

15. Find y' and y'' .

(a) $y = \cos(\sin(3\theta))$

(c) $y = e^{e^x}$

(d) $y = \frac{1}{(1 + \tan x)^2}$

(b) $y = \sqrt{1 - \sec t}$

16. Find the given derivative by finding the first few derivatives and observing the pattern that occurs.

(a) $\frac{d^{99}}{dx^{99}}(\sin x)$

(b) $\frac{d^{35}}{dx^{35}}(x \sin x)$

17. Find the derivative of the function. Simplify where possible.

(a) $y = (\tan^{-1} x)^2$

(c) $g(x) = \cos^{-1}(\sqrt{x})$

(e) $y = \arctan \sqrt{\frac{1-x}{1+x}}$

(b) $y = \sin^{-1}(2x + 1)$

(d) $y = \tan^{-1}(x - \sqrt{1+x^2})$

(f) $y = \cos^{-1}(\sin^{-1} t)$

18. Differentiate the function. In part (h), a is a constant.

(a) $f(x) = x \ln x - x$

(e) $f(x) = \log_{10} \sqrt{x}$

(h) $H(z) = \ln \sqrt{\frac{a^2 - z^2}{a^2 + z^2}}$

(b) $f(x) = \sin(\ln x)$

(f) $h(x) = \ln(x + \sqrt{x^2 - 1})$

(i) $y = \log_2(x \log_5 x)$

(c) $f(x) = \ln(\sin^2 x)$

(g) $G(y) = \ln \frac{(2y + 1)^5}{\sqrt{y^2 + 1}}$

(j) $y = \ln(\csc x - \cot x)$

(d) $g(x) = \ln(xe^{-2x})$

19. Find an equation of the tangent line to the curve at the given point.

(a) $y = \ln(x^2 - 3x + 1)$, $(3, 0)$

(c) $y = (\ln x)/x$, $(1, 0)$

(b) $y = x^2 \ln x$, $(1, 0)$

(d) $y = (\ln x)/x$, $(e, 1/e)$

20. Find y' and y'' .

(a) $y = \sqrt{x} \ln x$

(c) $y = \ln |\sec x|$

(b) $y = \frac{\ln x}{1 + \ln x}$

(d) $y = \ln(1 + \ln x)$

21. Use logarithmic differentiation to find the derivative of the function.

(a) $y = (x^2 + 2)^2(x^4 + 4)^4$

(d) $y = \sqrt{x}e^{x^2-x}(x+1)^{2/3}$

(h) $y = (\cos x)^x$

(b) $y = \frac{e^{-x} \cos^2 x}{x^2 + x + 1}$

(e) $y = x^{\sin x}$

(i) $y = \tan x)^{1/x}$

(c) $y = \ln \sqrt{\frac{x-1}{x^4+1}}$

(f) $y = x^x$

(j) $y = \sqrt{x}^x$

(g) $y = (\ln x)^{\cos x}$

22. Find a formula for $f^{(n)}(x)$ if $f(x) = \ln(x - 1)$.

23. Find $\frac{d^9}{dx^9}(x^8 \ln x)$

24. Use the definition of the derivative to prove that

$$\lim_{x \rightarrow 0} \frac{\ln(1+x)}{x} = 1$$

25. Show that

$$\lim_{n \rightarrow \infty} \left(1 + \frac{x}{n}\right)^n = e^x \text{ for any } x > 0.$$