Fact: Contractive sequences are Cauchy, and therefore convergent.

(1) Let $x_1 = 10$ and $x_{n+1} = 3 + \frac{2}{5}x_n$ for $n \geq 1$. Prove that (x_n) converges and find its limit.

(2) Use subsequences to prove that the sequence $x_n = \sin(n\pi/2)$ does not converge.

(3) Let $x_n = 5 + (2/5)^n$. Prove (x_n) is contractive.

(4) Let $y_n = \frac{1}{n}$. Prove (y_n) is not contractive.

(5) True or False. Provide a proof or counterexample.

- (a) Every Cauchy sequence converges.
- (b) Every convergent sequence is Cauchy.
- (c) Every contractive sequence converges.
- (d) Every convergent sequence is contractive.
- (e) Every bounded sequence converges.
- (f) Every monotone sequence converges.
- (g) Every bounded and monotone sequence converges.
- (h) Every convergent sequence is bounded.
- (i) Every convergent sequence is monotone.
- (j) There exists a convergent sequence with an unbounded subsequence.
- (k) There exists an unbounded sequence with a convergent subsequence.
- (l) If (a_n) and (b_n) are increasing, then $(a_n + b_n)$ is increasing.
- (m) If (a_n) and (b_n) are increasing, then $(a_n b_n)$ is increasing.
- (n) If (a_n) and (b_n) are nonnegative and increasing, then $(a_n b_n)$ is increasing.
- (o) If $|x_{n+2} - x_{n+1}| < |x_{n+1} - x_n|$ for all n, then (x_n) converges.