Section 1.7 - Integrals Resulting in Inverse Trigonometric Functions

Evaluate the following integrals.

(i) $\displaystyle\int_{\tan 1}^{\tan 8} \frac{dx}{x^2+1}$

(ii) $\displaystyle\int \frac{dx}{\sqrt{1-16x^2}}$

(iii) $\displaystyle\int \frac{dx}{x\sqrt{25x^2-1}}$

(iv) $\displaystyle\int \frac{(x+5)}{\sqrt{4-x^2}}\,dx$

(v) $\displaystyle\int \frac{\ln(\cos^{-1} x)}{(\cos^{-1} x)\sqrt{1-x^2}}\,dx$

(vi) $\displaystyle\int \frac{\tan^{-1} x}{1+x^2}\,dx$

(vii) $\displaystyle\int_0^4 \frac{dt}{4t^2+9}$

(viii) $\displaystyle\int_{-1}^{\sqrt{3}} \frac{dx}{\sqrt{4-25x^2}}$

(ix) $\displaystyle\int_{1/4}^{1/2} \frac{dx}{x\sqrt{16x^2-1}}$

(x) $\displaystyle\int \frac{dx}{\sqrt{5^{2x}-1}}$

Section 3.1 - Integration by Parts

(1) $\displaystyle\int x\cos 5x\,dx$

(2) $\displaystyle\int xe^{-x}\,dx$

(3) $\displaystyle\int re^{r/2}\,dr$

(4) $\displaystyle\int t\,\sin 2t\,dt$

(5) $\displaystyle\int x^2\,\sin \pi x\,dx$

(6) $\displaystyle\int \ln \sqrt[3]{x}\,dx$

(7) $\displaystyle\int \arctan 4t\,dt$

(8) $\displaystyle\int \sin^{-1} x\;dx$

(9) $\displaystyle\int e^{2\theta}\,\sin 3\theta\,d\theta$

(10) $\displaystyle\int e^{-\theta}\,\cos 2\theta\,d\theta$

(11) $\displaystyle\int_0^{\pi} t\,\sin 3t\,dt$

(12) $\displaystyle\int_0^1 (x^2+1)e^{-x}\,dx$

(13) $\displaystyle\int_1^2 \frac{\ln x}{x^2}\,dx$

(14) $\displaystyle\int_4^9 \frac{\ln y}{\sqrt{y}}\,dy$

(15) $\displaystyle\int_0^1 \frac{y}{e^{2y}}\,dy$

(16) $\displaystyle\int_1^{\sqrt{3}} \arctan(1/x)\,dx$

(17) $\displaystyle\int_0^{1/2} \cos^{-1} x\;dx$

(18) $\displaystyle\int_0^1 \frac{r^3}{\sqrt{4+r^2}}\,dr$

(19) $\displaystyle\int_1^2 (\ln x)^2\,dx$

(20) $\displaystyle\int_0^t e^s\,\sin(t-s)\,ds$

First make a substitution and then use integration by parts to evaluate the integral.

(21) $\displaystyle\int \cos\sqrt{x}\,dx$

(22) $\displaystyle\int t^3 e^{-t^2}\,dt$

(23) $\displaystyle\int_{\sqrt{\pi/2}}^{\sqrt{\pi}} \theta^3\,\cos(\theta^2)\,d\theta$

(24) $\displaystyle\int_0^{\pi} e^{\cos t}\,\sin 2t\,dt$

(25) $\displaystyle\int \sin(\ln x)\,dx$

(26) $\displaystyle\int x\ln(1+x)\,dx$

Use integration by parts to prove the reduction formula.

(27) $\displaystyle\int (\ln x)^n\,dx = x(\ln x)^n - n\int (\ln x)^{n-1} dx$

(28) $\displaystyle\int x^n e^x\,dx = x^n e^x - n\int x^{n-1} e^x dx$