(1) Suppose $\dfrac{x_n}{2+x_n}$ converges to 4. Prove that x_n must converge, and find its limit.

(2) Prove that if x_n converges to a nonzero limit and y_n diverges, then $x_n y_n$ diverges.

(3) Suppose $a_n \geq 0$ for all n.

   (a) Suppose $\lim a_n = 0$. Prove that $\lim \sqrt{a_n} = 0$.

   (b) Suppose $\lim a_n = a > 0$. Prove that $\lim \sqrt{a_n} = \sqrt{a}$.

   Hint: $(\sqrt{a_n} - \sqrt{a})(\sqrt{a_n} + \sqrt{a}) = a_n - a$.

(4) Let $a_1 = 2$ and $a_{n+1} = -\frac{1}{4}a_n + 7$. Assuming the sequence (a_n) converges, find its limit.

(5) Prove the Squeeze Theorem. That is, show that if $x_n \leq y_n \leq z_n$ for all n and $\lim\limits_{n\to\infty} x_n = \lim\limits_{n\to\infty} z_n = a$, then $\lim\limits_{n\to\infty} y_n = a$. (Note: This is not simply a matter of using the Order Limit Theorem since that assumes the existence of the limits involved. Here we are not assuming the sequence (y_n) converges.)

(6) Let $a_1 = -7$ and $a_{n+1} = \frac{3}{7}a_n - 2$. Prove (a_n) converges and find its limit.

(7) Let $b_1 = 2$ and $b_{n+1} = 3 - \frac{1}{b_n}$ for $n \geq 1$.

   (a) Prove that $1 < b_n < 3$ for all $n \geq 1$.

   (b) Prove that $b_{n+1} > b_n$ for all $n \geq 1$.

   (c) Prove (b_n) converges and find its limit.

(8) Define $c_1 = 1$ and $c_{n+1} = \frac{n^2 c_n}{n^2+1}$. Prove (c_n) converges. You do not need to find its limit.