(1) Prove that $[0,1] \sim [0,2) \cup [4,6]$.

(2) Prove that if $A \sim B$ and $C \sim D$, then $A \times C \sim B \times D$.

(3) (a) Let S_n be the set of all subsets of $\mathbb{N}$ with exactly n elements. Prove that S_n is countable. (You may use the result of Problem #5)

  (b) Let S be the set of all finite subsets of $\mathbb{N}$. Prove that S is countable.

(4) Prove that $(a,b) \sim (c,d)$ for any $a < b$ and $c < d$.

(5) Prove that if A_k is countable for $k = 1, \ldots, n$ then $A_1 \times A_2 \times \cdots \times A_n$ is countable.

(6) Let S be the set of all polynomials with rational coefficients. Prove that S is countable.

(7) Prove that $[0,1] \sim [0,1)$.

(8) Prove that $\mathbb{I} \sim \mathbb{R}$.

(9) Let A be any nonempty set and let B be the set of all functions $f : \{0,1\} \to A$. Show that $B \sim A \times A$.