

1. Evaluate the following limits.

$$(i) \lim_{x \rightarrow -2} 4x^2 - 5x + 7$$

$$(ii) \lim_{x \rightarrow 1} (x + 3)(x - 7)$$

$$(iii) \lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}$$

$$(iv) \lim_{x \rightarrow -3} \frac{\sqrt{x+7} - 2}{x+3}$$

$$(v) \lim_{x \rightarrow 1} \frac{x^2 - 2x + 3}{x - 1}$$

$$(vi) \lim_{x \rightarrow 2} \frac{(x-2)(x+3)}{\sqrt{x+2} - 2}$$

$$(vii) \lim_{x \rightarrow 1} \frac{\frac{1}{x} - 1}{x - 1}$$

$$(viii) \lim_{t \rightarrow 0} \frac{\frac{1}{(t+2)^2} - \frac{1}{4}}{t}$$

$$(ix) \lim_{h \rightarrow 0} \frac{(3+h)^2 - 9}{h}$$

$$(x) \lim_{h \rightarrow 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$(xi) \lim_{t \rightarrow 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$$

$$(xii) \lim_{t \rightarrow 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$$

$$(xiii) \lim_{x \rightarrow 0} |x|$$

$$(xiv) \lim_{x \rightarrow 0} \frac{|x|}{x}$$

2. Let

$$g(x) = \begin{cases} x + 1 & \text{if } x \neq 1 \\ \pi & \text{if } x = 1 \end{cases}$$

Find $\lim_{x \rightarrow 1} g(x)$ if it exists.

3. Show that $\lim_{x \rightarrow 0} x^2 \sin^2 \left(\frac{1}{x} \right) = 0$

4. Compute the limits.

$$(i) \lim_{x \rightarrow \infty} \frac{\sin x}{x}$$

$$(ii) \lim_{x \rightarrow \infty} \frac{2 - \cos x}{x + 3}$$

$$(iii) \lim_{x \rightarrow \infty} \frac{\cos^2(2x)}{3 - 2x}$$

5. Compute the limits.

$$(i) \lim_{x \rightarrow 0} \frac{\sin x}{7x}$$

$$(ii) \lim_{x \rightarrow 0} \frac{\sin 5x}{x}$$

$$(iii) \lim_{x \rightarrow 0} \frac{\sin 7x}{\sin 4x}$$

$$(iv) \lim_{\theta \rightarrow 0} \frac{1 - \cos 4\theta}{\theta}$$

$$(v) \lim_{\theta \rightarrow 0} \frac{\sin 3\theta}{\tan 2\theta}$$

$$(vi) \lim_{x \rightarrow 0} \sqrt{x^3 + x^2} \sin \left(\frac{\pi}{x} \right)$$

$$(vii) \lim_{x \rightarrow -6} \frac{2x + 12}{x + 6}$$

6. Prove that $\lim_{x \rightarrow 0} x^4 \cos \left(\frac{2}{x} \right) = 0$

7. If

$$4x - 9 \leq f(x) \leq x^2 - 4x + 7$$

for $x \geq 0$, find $\lim_{x \rightarrow 4} f(x)$.