

- (1) Compute without proofs, the suprema and infima of the following sets:
 - (a) $\{n \in \mathbb{N} : n^2 < 10\}$.
 - (b) $\{n/(m+n) : m, n \in \mathbb{N}\}$.
 - (c) $\{n/(2n+1) : n \in \mathbb{N}\}$.
 - (d) $\{n/m : m, n \in \mathbb{N} \text{ with } m+n \leq 10\}$.
 - (e) $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots\right\}$
- (2)
 - (a) Give an example of a set of rational numbers whose supremum is irrational.
 - (b) Give an example of a set of irrational numbers whose supremum is rational.
- (3) Suppose A and B are nonempty subsets of \mathbb{R} such that $\inf(B) < \sup(A)$. Show that there exist $a \in A$ and $b \in B$ such that $b < a$.
- (4) Let A and B be subsets of \mathbb{R} that are bounded below, and let $A+B = \{a+b : a \in A \text{ and } b \in B\}$. Prove that $\inf(A+B) = \inf(A) + \inf(B)$.
- (5) Let $A = \left\{\frac{3n+12}{n+2} : n \in \mathbb{N}\right\}$.
 - (a) Find $\min(A)$ and $\max(A)$, and prove your assertions.
 - (b) Find $\inf(A)$ and $\sup(A)$, and prove your assertions.