Definition

A series $\sum a_n$ is called **absolutely convergent** if the series of absolute values $\sum |a_n|$ is convergent.

Theorem

If a series $\sum a_n$ is absolutely convergent, then it is convergent.

Note It is possible for a series to be convergent but not absolutely convergent.

Definition

A series $\sum a_n$ is called **conditionally convergent** if $\sum a_n$ is convergent but the series of absolute values $\sum |a_n|$ is divergent.

(1) Determine whether the series converges absolutely, conditionally, or not at all.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}}$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^{n-1}}{(1.1)^n}$

(c) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{\cos n\pi}{(\ln n)^2}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n\, n^4}{n^3+1}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin(\frac{\pi n}{4})}{n^2}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{1+\frac{1}{n}}$

(h) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos n}{2^n}$

(2) Determine convergence or divergence by any method.

(a) $\displaystyle\sum_{n=0}^{\infty} 7^{-n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{7.5}}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{5^n - 3^n}$

(d) $\displaystyle\sum_{n=2}^{\infty} \frac{n}{n^2 - n}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{3n^4 + 12n}$

(f) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n^2+1}}$

(g) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

(h) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n n}{\sqrt{n^2+1}}$

(i) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n+1)!}$

(j) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{5^n}$

(k) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\, n^2\, e^{-n^3/3}$

(l) $\displaystyle\sum_{n=1}^{\infty} n\, e^{-n^3/3}$

(m) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n^{1/2}\,(\ln n)^2}$

(n) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n\,(\ln n)^{1/4}}$

(o) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{1.05}}$

(p) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{(\ln n)^2}$