Sections 5.1 - 5.6

(1) (a) What is a sequence?

(b) What does it mean to say that $\lim_{n\to\infty} a_n = 8$?

(c) What does it mean to say that $\lim_{n\to\infty} a_n = \infty$?

(2) (a) What is a convergent sequence? Give two examples.

(b) What is a divergent sequence? Give two examples.

(3) List the first six terms of the sequence defined by $a_n = \dfrac{n}{2n+1}$. Does the sequence appear to have a limit? If so, find it.

(4) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

(a) $\{1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}, \ldots\}$

(b) $\{1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \ldots\}$

(c) $\{2, 7, 12, 17, \ldots\}$

(d) $\{-\frac{1}{4}, \frac{2}{9}, -\frac{3}{16}, \frac{4}{25}, \ldots\}$

(e) $\{1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots\}$

(f) $\{5, 1, 5, 1, 5, 1, \ldots\}$

(5) Determine whether the sequence converges or diverges. If it converges, find the limit.

(a) $a_n = \dfrac{3+5n^2}{n+n^2}$

(b) $a_n = 1 - (0.2)^n$

(c) $a_n = e^{1/n}$

(d) $a_n = \tan\left(\dfrac{2n\pi}{1+8n}\right)$

(e) $a_n = \cos(2/n)$

(f) $a_n = \dfrac{(-1)^{n-1}\, n}{n^1+1}$

(g) $a_n = \dfrac{(-1)^n\, n^3}{n^3+2n^2+1}$

(h) $\left\{\dfrac{e^n+e^{-n}}{e^{2n}-1}\right\}$

(i) $a_n = \dfrac{\cos^2 n}{2^n}$

(j) $a_n = \dfrac{(\ln n)^2}{n}$

(6) Determine whether the sequence is increasing, decreasing, or not monotonic. Is the sequence bounded?

(a) $a_n = \dfrac{1}{2n+3}$

(b) $a_n = n(-1)^n$

(c) $a_n = \dfrac{2n-3}{3n+4}$

(d) $a_n = n + \dfrac{1}{n}$

(7) Show that the sequence defined by

$$a_1 = 1 \quad a_{n+1} = 3 - \frac{1}{a_n}$$

is increasing and $a_n < 3$ for all n. Deduce that $\{a_n\}$ is convergent and find its limit.

(8) (a) What is the difference between a sequence and a series?

(b) What is a convergent series? What is a divergent series?

(9) Explain what it means to say that $\displaystyle\sum_{n=1}^{\infty} a_n = 5$?

(10) Let $a_n = \dfrac{2n}{3n+1}$.

(a) Determine whether $\{a_n\}$ is convergent.

(b) Determine whether $\displaystyle\sum_{n=1}^{\infty} a_n$ is convergent.

(11) Determine whether the geometric series is convergent or divergent. If it is convergent, find its sum.

(a) $3 - 4 + \frac{16}{3} - \frac{64}{9} + \cdots$

(b) $10 - 2 + 0.4 - 0.08 + \cdots$

(c) $1 + 0.4 + 0.16 + 0.064 + \cdots$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{\pi^n}{3^{n+1}}$

(12) Determine whether the series is convergent or divergent. If it is convergent, find its sum.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{3n-1}$

(c) $\displaystyle\sum_{k=2}^{\infty} \frac{k^2}{k^2-1}$

(d) $\displaystyle\sum_{n=1}^{\infty} \arctan$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1+3^n}{2^n}$

(13) Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{2}{n^2-1}$

(b) $\displaystyle\sum_{n=1}^{\infty} \ln \frac{n}{n+1}$

(14) Find the values of x for which the series converges. Find the sum of the series for those values of x.

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(x+3)^n}{2^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3^n}$

(15) If the n-th partial sum of a series $\displaystyle\sum_{n=1}^{\infty} a_n$ is

$$s_n = \frac{n-1}{n+1},$$

find a_n and $\displaystyle\sum_{n=1}^{\infty} a_n$.

(16) Use the Integral Test to determine whether the series is convergent or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^5}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n+4}}$

(17) Use the Comparison Test to determine whether the series is convergent or divergent.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

(b) $\displaystyle\sum_{n=2}^{\infty} \frac{n^3}{n^4-1}$

(18) Find the values of p for which the following series is convergent.

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^p}$$

(19) Determine whether the series is convergent or divergent.

(a) $\sum_{n=1}^{\infty} \frac{2}{n^{0.85}}$

(b) $\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

(c) $\sum_{n=1}^{\infty} \frac{1}{n^2+9}$

(d) $\sum_{n=1}^{\infty} \frac{n^2-5n}{n^3+n+1}$

(e) $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$

(f) $\sum_{n=1}^{\infty} ne^{-n}$

(g) $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2+1}$

(h) $\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$

(i) $\sum_{n=1}^{\infty} \frac{4+3^n}{2^n}$

(j) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3+1}}$

(20) Test the series for convergence or divergence.

(a) $\frac{4}{7} - \frac{4}{8} + \frac{4}{9} - \frac{4}{10} + \frac{4}{11} - \cdots$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{2n+1}$

(c) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$

(d) $\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{\pi}{n}\right)$

(21) Determine whether the series is absolutely convergent.

(a) $\sum_{n=1}^{\infty} \frac{(-3)^n}{n^3}$

(b) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\sqrt{n}}{n+1}$

(c) $\sum_{n=1}^{\infty} \frac{n!}{100^n}$

(d) $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$