(1) Let $x_1 = -7$ and $x_{n+1} = \frac{3}{7}x_n - 2$.
  (a) Prove that $x_n < -7/2$ for all $n \geq 1$.
  (b) Prove that $x_{n+1} > x_n$ for all $n \geq 1$.

(2) Let $c, r \in \mathbb{R}$ and assume $r \neq 1$. Prove that $\displaystyle\sum_{k=0}^{n} cr^k = \frac{c(1-r^{n+1})}{1-r}$ for any integer $n \geq 1$.

(3) For each $n \in \mathbb{N}$, every set of n real numbers has a largest element.

(4) Define $a_1 = 4$ and $a_{n+1} = \frac{1}{6}a_n^2 + 1$ for $n \geq 1$. Prove that $a_n > 1$ for all n and that $a_{n+1} < a_n$ for all integers $n \geq 1$.

(5) Prove that there exists a real number β such that $\beta^2 = 2$.

(6) For any $x, y \in \mathbb{R}$, prove the following:
  (a) If $x + y = x$, then $y = 0$
  (b) If $x + y = 0$, then $y = -x$
  (c) If $x \neq 0$ and $x \cdot y = x$, then $y = 1$
  (d) If $x \neq 0$ and $x \cdot y = 1$, then $y = 1/x$
  (e) If $x \neq 0$ and $y \neq 0$, then $x \cdot y \neq 0$
  (f) $\left|\dfrac{x}{y}\right| = \dfrac{|x|}{|y|}$ if $y \neq 0$
  (g) $-|x| \leq x \leq |x|$
  (h) $|x| \leq z \Leftrightarrow -z \leq x \leq z$
  (i) $||y| - |x|| \leq |y - x|$

(7) For any positive integer n and any real numbers x_1 through x_n, $\left|\displaystyle\sum_{k=1}^{n} x_k\right| \leq \displaystyle\sum_{k=1}^{n} |x_k|$.

(8) Prove that the set of irrational numbers is dense in $\mathbb{R}$. (Hint: Given $a < b$, apply the density of the set of rationals to the interval $(a - \sqrt{2}, b - \sqrt{2})$.)

(9) Suppose $a > b > 0$. Prove that $\sqrt{a} > \sqrt{b}$.

(10) Prove that for every real number x, there exists an integer n such that $n \leq x < n + 1$. We call n the **greatest integer** of x and write $n = \lfloor x \rfloor$.