Section 2.4 - Arc length of a curve

(1) Express the arc length of the curve $y = x^4$ between $x = 2$ and $x = 6$ as an integral (but do not evaluate).

(2) Express the arc length of the curve $y = \tan x$ for $0 \le x \le \frac{\pi}{4}$ as an integral (but do not evaluate).

(3) Find the arc length of $y = \frac{1}{12}x^3 + x^{-1}$ for $1 \le x \le 2$. Hint: Show that $1 + (y')^2 = \left(\frac{1}{4}x^2 + x^{-2}\right)^2$.

(4) Find the arc length of $y = \left(\frac{x}{2}\right)^4 + \frac{1}{2x^2}$ over $[1, 4]$. Hint: Show that $1 + (y')^2$ is a perfect square.

(5) Find the exact length of the curve.

$$x = \frac{1}{3}\sqrt{y}\,(y - 3), \quad 1 \le y \le 9$$

(6) Find the exact length of the curve.

$$y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad [1, 2e]$$

(7) A hawk flying at 15 m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation

$$y = 180 - \frac{x^2}{45}$$

until it hits the ground, where y is its height above the ground and x is the horizontal distance traveled in meters. Calculate the distance traveled by the prey from the time it is dropped until the time it hits the ground. Express your answer correct to the nearest tenth of a meter.

(8) For the function $f(x) = \frac{1}{4}e^x + e^{-x}$, show that the arc length on the interval $[0, 1]$ has the same value as the area under the curve. Is it true for any interval $[a, b]$?

(9) Find the length of the curve

$$y = \int_1^x \sqrt{t^3 - 1}\,dt \quad 1 \le x \le 4$$

(10) Calculate the length of the astroid $x^{2/3} + y^{2/3} = 1$.