(1) List the ordered pairs in the relation R from $A = \{0, 1, 2, 3, 4\}$ to $B = \{0, 1, 2, 3\}$, where $(a, b) \in R$ if and only if

(a) $a = b$
(b) $a + b = 4$
(c) $a > b$
(d) $a \mid b$
(e) $\gcd(a, b) = 1$
(f) $\operatorname{lcm}(a, b) = 2$

(2) (a) Is the "divides" relation on the set of positive integers reflexive?
(b) Is the "divides" relation on the set of positive integers transitive?

(3) Determine whether the relation R on the set of all people is reflexive, symmetric, antisymmetric, and/or transitive, where $(a, b) \in R$ if and only if
(a) a is taller than b.
(b) a and b were born on the same day.
(c) a has the same first name as b.
(d) a and b have a common grandparent.

(4) Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if
(a) $x + y = 0$
(b) $x = \pm y$
(c) $x - y$ is a rational number
(d) $x = 2y$
(e) $xy > 0$
(f) $xy = 0$
(g) $x = 1$
(h) $x = 1$ or $y = 1$

(5) A relation R on the set A is **irreflexive** if for every $a \in A, (a, a) \notin R$. That is, R is irreflexive if no elements in A is related to itself.
(a) Which relations in Exercise 3 are irreflexive?
(b) Which relations in Exercise 4 are irreflexive?

(6) A relation R is called **asymmetric** if $(a, b) \in R$ implies that $(b, a) \notin R$.
(a) Which relations in Exercise 3 are asymmetric?
(b) Which relations in Exercise 4 are asymmetric?

(7) Give an example of a relation on a set that is
(a) both symmetric and antisymmetric.
(b) neither symmetric nor antisymmetric.

(8) How many relations are there on a set with n elements that are

(a) symmetric?
(b) antisymmetric?
(c) asymmetric?
(d) irreflexive?
(e) reflexive and symmetric?
(f) neither reflexive nor irreflexive?

(9) Consider the following relations on the set of real numbers:

$$R_1 = \{(a, b) \in \mathbb{R}^2 \mid a > b\}, \text{ the "greater than" relation}$$
$$R_2 = \{(a, b) \in \mathbb{R}^2 \mid a \geq b\}, \text{ the "greater than or equal to" relation}$$

Find

(a) $R_1 \setminus R_2$
(b) $R_2 \setminus R_1$
(c) $R_1 \circ R_1$
(d) $R_1 \circ R_2$
(e) $R_2 \circ R_2$
(f) $R_2 \circ R_1$

(10) Let R be the relation $R = \{(a, b) \mid a \text{ divides } b\}$ on the set of positive integers. Find R^{-1}.

(11) Let R be the relation on the set of real numbers such that xRy if and only if x and y are real numbers that differ by less than 1, that is $|x-y|<1$. Show that R is not an equivalence relation.

(12) Show that the "divides" relation is the set of positive integers in not an equivalence relation.

(13) Let R be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d)) \in R$ if and only if $a+d=b+c$. Show that R is an equivalence relation.

(14) Let R be the relation on the set of ordered pairs of positive integers such that $((a,b),(c,d)) \in R$ if and only if $ad=bc$. Show that R is an equivalence relation.

(15) (a) Show that the relation R on the set of all differentiable functions from $\mathbb{R}$ to $\mathbb{R}$ consisting of all pairs (f,g) such that $f'(x)=g'(x)$ for all real numbers x is an equivalence relation.
(b) Which functions are in the same equivalence class as the function $f(x)=x^2$?

(16) Let n be a positive integer. For integers, a,b we define $a \sim b$ if $n\,|\,(a-b)$. Show that $\sim$ is an equivalence relation on the set of integers.

(17) (a) On $\mathbb{R}^2$, define $(a_1,a_2) \sim (b_1,b_2)$ if $a_1^2+a_2^2=b_1^2+b_2^2$. Is $\sim$ an equivalence relation?
(b) Let A be the set of all integers and let B be the set of all nonzero integers. On the set $S=A\times B$ of ordered pairs, define $(m,n)\sim(p,q)$ if $mq=np$. Is $\sim$ an equivalence relation?

(18) For each of the following relations on $\mathbb{R}$, determine which of the three conditions of an equivalence relation hold.
(a) For $a,b\in\mathbb{R}$, define $a\sim b$ if $a-b\in\mathbb{Q}$, where $\mathbb{Q}$ is the set of rational numbers.
(b) For $a,b\in\mathbb{R}$, define $a\sim b$ if $|a-b|\le 1$.

(19) In $\mathbb{R}^3$, consider the standard (x,y,z)-coordinate system. We can define a partition of $\mathbb{R}^3$ by using planes parallel to the (x,y)-plane. Describe the corresponding equivalence relation by giving conditions on the coordinates x,y,z.

(20) Let S be a set and let $2^S=\{A|A\subseteq S\}$ be the collection of all subsets of S. Define $\sim$ on 2^S by letting $A\sim B$ if and only if there exists a one-to-one correspondence from A to B.
(a) Show that $\sim$ is an equivalence relation on 2^S.
(b) If $S=\{1,2,3,4\}$, list the elements of 2^S and find each equivalence class determined by $\sim$.