

- (1) Translate the given statement into propositional logic using the propositions provided.
- (i) You cannot edit a protected Wikipedia entry unless you are an administrator.
 e : You can edit a protected Wikipedia entry
 a : You are an administrator
- (ii) You can see the movie only if you are over 18 years old or you have the permission of a parent.
 m : You can see the movie
 e : You are over 18 years old
 p : You have the permission of a parent.
- (2) Express these system specifications using the propositions p *The message is scanned for viruses* and q *The message was sent from an unknown system* together with logical connectives (including negations).
- (a) The message is scanned for viruses whenever the message was sent from an unknown system.
 (b) The message was sent from an unknown system but it was not scanned for viruses.
 (c) It is necessary to scan the message for viruses whenever it was sent from an unknown system.
 (d) When a message is not sent from an unknown system it is not scanned for viruses.
- (3) Let $Q(x, y)$ denote the statement “ x is a capital of y ”. What are these truth values?
- (i) $Q(\text{Denver, Colorado})$ (iii) $Q(\text{Boston, Massachusetts})$
 (ii) $Q(\text{Detroit, Michigan})$ (iv) $Q(\text{New York, New York})$
- (4) Let $P(x)$ be the statement “ x spends more than five hours every weekday in class,” where the domain for x consists of all students. Express each of these quantifications in English.
- (i) $\exists x P(x)$ (iii) $\exists x \neg P(x)$
 (ii) $\forall x P(x)$ (iv) $\forall x \neg P(x)$
- (5) Let $N(x)$ be the statement “ x has visited North Dakota,” where the domain consists of the students in your school. Express each of these quantifications in English.
- (i) $\exists x N(x)$
 (ii) $\forall x N(x)$
 (iii) $\neg \exists x N(x)$
 (iv) $\exists x \neg N(x)$
 (v) $\neg \forall x N(x)$
 (vi) $\forall x \neg N(x)$
- (6) Translate these statements into English, where $C(x)$ is “ x is a comedian ” and $F(x)$ is “ x is funny ” and the domain consists of all people.
- (i) $\forall x (C(x) \rightarrow F(x))$ (iii) $\exists x (C(x) \rightarrow F(x))$
 (ii) $\forall x (C(x) \wedge F(x))$ (iv) $\exists x (C(x) \wedge F(x))$

- (7) Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.
- (i) No one is perfect.
 - (ii) Not everyone is perfect.
 - (iii) All your friends are perfect.
 - (iv) At least one of your friends is perfect.
- (8) Let $P(x)$ be the statement " $x = x^2$ ". If the domain consists of the integers, what are these truth values?
- (i) $P(0)$
 - (ii) $P(1)$
 - (iii) $P(2)$
 - (iv) $P(-1)$
 - (v) $\exists x P(x)$
 - (vi) $\forall x P(x)$
- (9) Determine the truth value of each of these statements if the domain consists of all integers.
- (i) $\forall n(n + 1 > n)$
 - (ii) $\exists n(2n = 3n)$
 - (iii) $\exists n(n = -n)$
 - (iv) $\forall n(3n \leq 4n)$
- (10) For each of these statements find a domain for which the statement is true and a domain for which the statement is false.
- (i) Everyone is studying discrete mathematics.
 - (ii) Everyone is older than 21 years.
 - (iii) Every two people have the same mother.
 - (iv) No two different people have the same grandmother.
- (11) Express the negation of these propositions using quantifiers, and then express the negation in English.
- (i) Some drivers do not obey the speed limit.
 - (ii) All Swedish movies are serious.
 - (iii) No one can keep a secret.
 - (iv) There is someone in this class who does not have a good attitude.
- (12) Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.
- (i) $\forall x(x^2 \neq x)$
 - (ii) $\forall x(x^2 \neq 2)$
 - (iii) $\forall x(|x| \geq 0)$