MATH 243 - 01 Mathematical Structures **Quiz 3** **February 17, 2023**

Your Name:

Duration of the Quiz is 25 minutes. There are five problems, worth 20 points. Show all your work for full credit. Books, notes etc. are prohibited. Calculators are NOT permitted.

1. Let $Q(x, y)$ denote "$x + y = 0$". The domain for all variables consists of all real numbers.

   (i) The quantification $\exists y \, \forall x \, Q(x, y)$ denotes the proposition "There is a real number y such that for every real number x, $Q(x, y)$." What is the truth value of the quantification $\exists y \, \forall x \, Q(x, y)$?

   (ii) The quantification $\forall x \, \exists y \, Q(x, y)$ denotes the proposition "For every real number x there is a real number y such that $Q(x, y)$." What is the truth value of the quantification $\forall x \, \exists y \, Q(x, y)$?

2. For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

   (i) the set of people who speak English, the set of people who speak English with an Australian accent.

   (ii) the set of fruits, the set of citrus fruits.

3. Determine whether each of these statements is true or false.

   (a) $\emptyset \in \{\emptyset\}$

   (b) $\emptyset \in \{\emptyset, \{\emptyset\}\}$

   (c) $\{\emptyset\} \in \{\{\emptyset\}\}$

4. Determine the truth value of the statement $\forall x \, \exists y \, (xy = 1)$ if the domain for the variables consists of

   (a) the non zero real numbers.

   (b) the nonzero integers.

   (c) the positive real numbers.

5. Show that the conditional statement $\neg p \to (p \to q)$ is a tautology

   (a) without using a truth table.

   (b) by using a truth table.