MATH 243 - 01 Mathematical Structures **Midterm Exam 2** **April 12, 2022**

Your Name:

Duration of the exam is 90 minutes. There are six problems, worth 50 points. Show all your work for full credit. Books, notes etc. are prohibited. Calculators are NOT permitted.

1. (a) (3 points) Find a formula for

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n}$$

by examining the values of this expression for small values of n.

(b) (5 points) Use mathematical induction to prove the formula you conjectured in part (a).

2. (12 points) Prove that if n is an integer, these four statements are equivalent:
(i) n^2 is odd, (ii) $1 - n$ is even, (iii) n^3 is odd, (iv) $n^2 + 1$ is even.

3. Let R be the relation on $\mathbb{Z}$ defined by

$$R = \{(a, b) \mid a \equiv b \pmod{5}\}.$$

(a) (6 points) Prove that R is an equivalence relation.

(b) (2 points) Give three different integers that are in the equivalence class $[\,3\,]$.

(c) (2 points) Let R be the relation defined above, and $a, b \in \mathbb{Z}$. If $[a] \cap [b] \neq \emptyset$, what can you conclude about a and b?

4. (a) (3 points) Find the prime factorization of 3780.

(b) (3 points) Find $\phi(34)$.

(c) (2 points) What is the least common multiple of the following pair of integers?

$$3^7 \cdot 5^3 \cdot 7^3,\ 2^{11} \cdot 3^5 \cdot 5^9$$

5. (a) (3 points) Use the Euclidean Algorithm to find $\gcd(1001, 1331)$.

(b) (3 points) Express $\gcd(1001, 1331)$ as a linear combination of 1001 and 1331.

6. (a) (2 points) What are the quotient and remainder when -123 divided by 19?

(b) (2 points) Suppose that a and b are integers, $a \equiv 11 \pmod{19}$, and $b \equiv 3 \pmod{19}$. Find the integer c with $0 \le c \le 18$ such that $c \equiv a^3 + 4b^3 \pmod{19}$.

(c) (2 points) Determine whether 179 is prime. **Hint:** $\sqrt{179} \approx 13.37$

WORKSHEET