MATH 243 - 01 Mathematical Structures **Midterm Exam 1** **March 1, 2022**

Your Name:

Duration of the exam is 90 minutes. There are ten problems, worth 50 points. Show all your work for full credit. Books, notes etc. are prohibited. Calculators are NOT permitted.

1. Which of these sentences are propositions? What are the truth values of those that are propositions?

   (a) The earth is round.

   (b) $2 + 3 = 5$

   (c) Do you speak English?

   (d) $3 - x = 5$

   (e) The sun will come out tomorrow.

2. Write the following statements using quantifiers, and determine the truth value of each statement if the domain for x is the set of real numbers and domain for y is the set of integers.

   (a) For every x, there is a y such that $x = 2y$.

   (b) For every y, there is an x such that $x = 2y$.

   (c) For every x and for every y, it is the case that $x = 2y$.

   (d) There exists an x such that for some y the equality $x = 2y$ holds.

   (e) There exists an x and a y such that $x = 2y$.

3. Let p and q be the propositions

   p: It is below freezing
   q: It is snowing

   Write these statements using p, q, and r and logical connectives (including negations).

   (a) It is below freezing and snowing.

   (b) It is below freezing but not snowing.

   (c) It is not below freezing and it is not snowing.

   (d) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.

   (e) That it is below freezing is necessary and sufficient for it to be snowing.

4. Determine whether each of following statements is true or false.

   (a) If $1 + 1 = 3$, then unicorns exist.

   (b) If $1 + 1 = 2$, then dogs can fly.

   (c) $\exists!\, x\, (x^2 + 2x = 15)$

   (d) $\exists y\, \forall x\ Q(x, y)$, where $Q(x, y)$ is the statement "$x + y = x - y$", and the domain for both variables consists of all integers.

   (e) $\exists n\, \forall m\, (nm = m)$, where the domain for all variables consists of all integers.

5. State the converse, contrapositive, and inverse of the following conditional statement.

I go to the beach whenever it is a sunny summer day.

6. Construct a truth table for the following compound proposition.

$$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$$

7. Prove that the following compound statement is a tautology without using a truth table.

$$(\neg q \wedge (p \to q)) \to \neg p$$

8. Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,

(a) $A_i = [-i, i]$, that is, the set of real numbers x with $-i \leq x \leq i$.

(b) $A_i = [i, \infty)$, that is, the set of real numbers x with $x \geq i$.

9. Prove that $A \setminus (A \setminus B) \subseteq B$.

10. Prove that $(A \cap B) \cup (A \cap \overline{B}) = A$.

WORKSHEET