Due by 8:05am on Friday, March 17

(1) What is wrong with this proof?

Theorem 0.1. *If n^2 is positive, then n is positive.*

Proof. Suppose that n^2 is positive. Because the conditional statement "If n is positive, then n^2 is positive" is true, we can conclude that n is positive. □

(2) What is wrong with this proof?

Theorem 0.2. *If n is not positive, then n^2 is not positive.*

Proof. Suppose that n^2 is positive. Because the conditional statement "If n is positive, then n^2 is positive" is true, we can conclude that n is positive. □

(3) Is the following argument correct? It supposedly shows that n is an even integer whenever n^2 is an even integer.

Suppose that n^2 is even. Then $n^2 = 2k$ for some integer k. Let $n = 2l$ for some integer l. This shows that n is even.

(4) Prove or disprove that the product of two irrational numbers is irrational.

(5) Prove that if x is irrational, then $1/x$ is irrational.

(6) Show that if n is an integer and $n^3 + 5$ is odd, then n is even using
(a) a proof by contraposition.
(b) a proof by contradiction.

(7) Prove that if n is an integer and $3n + 2$ is even, then n is even using
(a) a proof by contraposition.
(b) a proof by contradiction.

(8) Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.

(9) Use a direct proof to show that every odd integer is the difference of two squares.

(10) Prove that if n is a perfect square, then $n + 2$ is not a perfect square.

(11) Use a proof by contraposition to show that if $x + y \geq 2$, where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

(12) Prove the proposition $P(0)$, where $P(n)$ is the proposition "If n is a positive integer greater than 1, then $n^2 > n$." What kind of proof did you use?

(13) Let $P(n)$ be the proposition "If a and b are positive real numbers, then $(a+b)^n \geq a^n + b^n$." Prove that $P(1)$ is true. What kind of proof did you use?

(14) Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.

(15) Show that these statements about the integer x are equivalent:
(i) $3x + 2$ is even, (ii) $x + 5$ is odd, (iii) x^2 is even.