Due by 4pm on Friday, February 24

(1) Find A^2 if
 (a) $A = \{0, 1, 3\}$
 (b) $A = \{1, 2, a, b\}$

(2) Prove that $\mathcal{P}(A) \subseteq \mathcal{P}(B)$ if and only if $A \subseteq B$

(3) Translate each of these quantifications into English and determine its truth value.

 (a) $\forall x \in \mathbb{R}\,(x^2 \neq -1)$
 (b) $\exists x \in \mathbb{Z}\,(x^2 = 2)$
 (c) $\forall x \in \mathbb{Z}\,(x^2 > 0)$
 (d) $\exists x \in \mathbb{R}\,(x^2 = x)$

(4) Let A be a set. Show that $\emptyset \times A = A \times \emptyset = \emptyset$

(5) Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$

(6) Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets.

 (a) $A \cap B$
 (b) $A \cup B$
 (c) $A \setminus B$
 (d) $B \setminus A$

(7) Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$. Find

 (a) $A \cap B$
 (b) $A \cup B$
 (c) $A \setminus B$
 (d) $B \setminus A$

(8) Assume that A is a subset of some underlying universal set U.
 (a) Prove the identity laws
 (b) Prove the domination laws
 (c) Prove the complement laws
 (d) Prove the second De Morgan's law by showing that if A and B are sets, then $\overline{A \cup B} = \overline{A} \cap \overline{B}$.

(9) Show that if A and B are sets with $A \subseteq B$, then
 (a) $A \cup B = B$
 (b) $A \cap B = A$

(10) Find $\displaystyle\bigcup_{i=1}^{\infty} A_i$ and $\displaystyle\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,

 (a) $A_i = \{i,\, i+1,\, i+2,\, \ldots\}$
 (b) $A_i = \{0, i\}$
 (c) $A_i = (0, i)$, that is, the set of real numbers x with $0 < x < i$.
 (d) $A_i = (i, \infty)$, that is, the set of real numbers x with $x > i$.