(1) Let n be a positive integer. Show that $f_Y(y) = (n+2)(n+1)y^n(1-y)$, $0 \le y \le 1$, is a pdf.

(2) If Y is an exponential random variable, $f_Y(y) = \lambda e^{-\lambda y}$, $y \ge 0$, ffnd $F_Y(y)$.

(3) Let $f_Y(y) = \frac{3}{2}y^2$, $-1 \le y \le 1$. Find $P\Big(|Y - \frac{1}{2}| < \frac{1}{4}\Big)$.

(4) The length of time, Y, that a customer spends in line at a bank teller's window before being served is described by the exponential pdf $f_Y(y) = 0.2e^{-0.2y}$, $y \ge 0$.

(a) What is the probability that a customer will wait more than ten minutes?
(b) Suppose the customer will leave if the wait is more than ten minutes. Assume that the customer goes to the bank twice next month. Let the random variable X be the number of times the customer leaves without being served. Calculate $p_X(1)$.

(5) Suppose $F_Y(y) = \frac{1}{12}(y^2 + y^3)$, $0 \le y \le 2$. Find $f_Y(y)$.

(6) If the pdf for Y is

$$f_Y(y) = \begin{cases} 0 & \text{if } |y| > 1 \\ 1 - |y| & \text{if } |y| \le 1 \end{cases}$$

find and graph $F_Y(y)$.

(7) A random variable Y has cdf

$$F_Y(y) = \begin{cases} 0 & y < 1 \\ \ln y & 1 \le y \le e \\ 1 & e < y \end{cases}$$

Find

(a) $P(Y < 2)$ (b) $P(2 < Y \le 2.5)$ (c) $P(2 < Y < 2.5)$ (d) $f_Y(y)$

(8) Suppose $f_Y(y) = 4y^3$, $0 \le y \le 1$. Find $P\Big(0 \le Y \le \frac{1}{2}\Big)$. Find the cdf for the random variable Y and calculate $P\Big(0 \le Y \le \frac{1}{2}\Big)$ using $F_Y(y)$. Define $W = 2Y$. Find $f_W(w)$. For which values of w is $f_W(w) \ne 0$?

(9) One continuous pdf that has a number of interesting applications in physics is the *Rayleigh distribution*, where the pdf is given by

$$f_Y(y) = \frac{y}{a^2}\, e^{-y^2/2a^2}, \quad a > 0;\ \ 0 \le y < \infty$$

Calculate the expected value for a random variable having a Rayleigh distribution.

(10) Find the median for each of the following pdfs:

(a) $f_Y(y) = (\theta + 1)y^\theta$, $0 \le y \le 1$, where $\theta > 0$. (b) $f_Y(y) = y + \frac{1}{2}$, $0 \le y \le 1$.

(11) Find the expected value of a RV Y that has a uniform distribution.

(12) In one of the early applications of probability to physics, James Clerk Maxwell, (1831 - 1879) showed that the speed S of a molecule in a perfect gas has a density function given by

$$f_S(s) = 4\sqrt{\frac{a^3}{\pi}}\, s^2\, e^{-as^2}, \quad s > 0,$$

where a is a constant depending on the temperature of the gas and the mass of the particle. What is the average *energy* of a molecule in a perfect gas?

(13) A random variable Y is described by the pdf

$$f_Y(y) = 2y, \ \ 0 \le y \le 1$$

What is the standard deviation of $3Y + 2$?

(14) Let Y be the random variable in Problem 1. Find Var(Y). For any positive integer k, find the kth moment around the origin.

(15) Suppose that the random variable Y is described by the pdf $f_Y(y) = c \cdot y^{-6}$, $y > 1$. Find c. What is the highest moment of Y that exists?