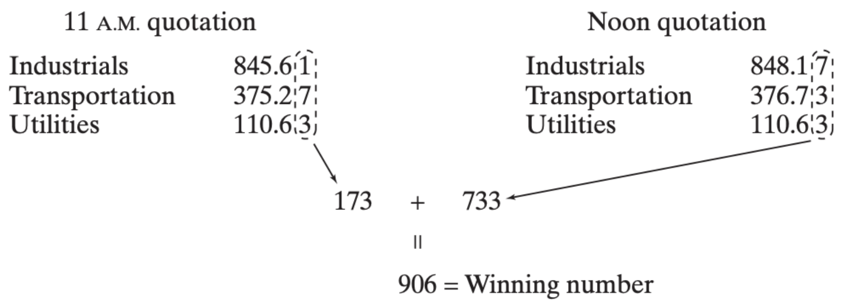
(1) Among the more common versions of the "numbers" racket is a game called D.J., its name deriving from the fact that the winning ticket is determined from Dow Jones averages. Three sets of stocks are used: Industrials, Transportations, and Utilities. Traditionally, the three are quoted at two different times, 11 a.m. and noon. The last digits of the earlier quotation are arranged to form a three-digit number; the noon quotation generates a second three-digit number, formed the same way. Those two numbers are then added together and the last three digits of that sum become the winning pick. The following figure shows a set of quotations for which 906 would be declared the winner.



The payoff in D.J. is 700 to 1. Suppose that we bet \$5. How much do we stand to win, or lose, on the average?

(2) Suppose that fifty people are to be given a blood test to see who has a certain disease. The obvious laboratory procedure is to examine each person's blood individually, meaning that fifty tests would eventually be run. An alternative strategy is to divide each person's blood sample into two parts—say, A and B. All of the A's would then be mixed together and treated as one sample. If that "pooled" sample proved to be negative for the disease, all fifty individuals must necessarily be free of the infection, and no further testing would need to be done. If the pooled sample gave a positive reading, of course, all fifty B samples would have to be analyzed separately. Under what conditions would it make sense for a laboratory to consider pooling the fifty samples?

(3) Consider the following game. A fair coin is flipped until the first tail appears; we win \$2 if it appears on the first toss, \$4 if it appears on the second toss, and, in general, $\$2^k$ if it first occurs on the kth toss. Let the random variable X denote our winnings. How much should we have to pay in order for this to be a fair game? [Note: A fair game is one where the difference between the ante and $E(X)$ is 0.]

(4) A certain type of tree has seedlings randomly dispersed in a large area, with the mean density of seedlings being approximately five per square yard. If a forester randomly locates ten 1-square-yard sampling regions in the area, find the probability that none of the regions will contain seedlings.

(5) Suppose that Y possesses a binomial distribution with $n = 20$ and $p = .1$. Find the exact value of $P(Y \leq 3)$ using the table of binomial probabilities given in Table 1 in Appendix of the textbook. Use Table 3, Appendix 3, to approximate this probability, using a corresponding probability given by the Poisson distribution. Compare the exact and approximate values for $P(Y \leq 3)$.

(6) Industrial accidents occur according to a Poisson process with an average of three accidents per month. During the last two months, ten accidents occurred. Does this number seem highly improbable if the mean number of accidents per month, μ, is still equal to 3? Does it indicate an increase in the mean number of accidents per month?

(7) The number of typing errors made by a typist has a Poisson distribution with an average of four errors per page. If more than four errors appear on a given page, the typist must retype the whole page. What is the probability that a randomly selected page does not need to be retyped?

(8) The random variable Y has a Poisson distribution and is such that $p(0) = p(1)$. What is $p(2)$?

(9) Customers arrive at a checkout counter in a department store according to a Poisson distribution at an average of seven per hour. What is the probability that exactly two customers arrive in the two-hour period of time between
   (a) 2:00 P.M. and 4:00 P.M. (one continuous two-hour period)?
   (b) 1:00 P.M. and 2:00 P.M. or between 3:00 P.M. and 4:00 P.M. (two separate one-hour periods that total two hours)?

(10) Approximately 4% of silicon wafers produced by a manufacturer have fewer than two large flaws. If Y , the number of flaws per wafer, has a Poisson distribution, what proportion of the wafers have more than five large flaws? [Hint: Use Table 3, Appendix 3 in the textbook]

(11) Let X be a random variable with pdf $p_X(k) = 1/n$, for $k = 0, 1, 2, \ldots, n-1$ and 0 otherwise. Show that $M_X(t) = \frac{1-e^{nt}}{n(1-e^t)}$.

(12) Find the expected value of e^{3X} if X is a binomial random variable with $n = 10$ and $p = \frac{1}{3}$.

(13) Which pdfs would have the following moment- generating functions?

(a) $M_X(t) = \left(\frac{1}{2} + \frac{1}{2}e^t\right)^4$

(b) $M_X(t) = 0.3e^t/(1 - 0.7e^t)$

(14) Find the moment-generating function for a Poisson random variable.

(15) Two chips are drawn at random and without replacement from an urn that contains five chips, numbered 1 through 5. If the sum of the chips drawn is even, the random variable X equals 5; if the sum of the chips drawn is odd, $X = -3$. Find the moment-generating function for X.