Distribution of a Random Variable

(1) Suppose we roll two fair six-sides dice, and let Y be the sum of the two numbers showing.

   (a) Compute $P(Y = y)$ for every real y.
   (b) Write a formula for $P(Y \in B)$, for any subset B of the real numbers.

(2) Consider flipping two independent fair coins. Let X be the number of heads that appear. Compute $P(X = x)$ for all real numbers x.

(3) Suppose we flip three fair coins, and let X be the number of heads showing.

   (a) Compute $P(X = x)$ for every real x.
   (b) Write a formula for $P(X \in B)$, for any subset B of the real numbers.

(4) Suppose we roll one fair six-sided die, and let Z be the number showing. Let $W = Z^3 + 4$, and let $V = \sqrt{Z}$.

   (a) Compute $P(W = w)$ for every real w.
   (b) Compute $P(V = v)$ for every real v.
   (c) Compute $P(ZW = x)$ for every real x.
   (d) Compute $P(VW = y)$ for every real y.
   (e) Compute $P(V + W = r)$ for every real r.

(5) Suppose that a bowl contains 100 chips: 30 are labelled 1, 20 are labelled 2, and 50 are labelled 3. The chips are thoroughly mixed, a chip is drawn, and the number X on the chip is noted.

   (a) Compute $P(X = x)$ for every real x.
   (b) Suppose the first chip is replaced, a second chip is drawn, and the number Y on the chip noted. Compute $P(Y = y)$ for every real number y.
   (c) Compute $P(W = w)$ for every real number when w when $W = X + Y$.

(6) Suppose a standard deck of 52 playing cards is thoroughly shuffled and a single card is drawn. Suppose an ace has value 1, a jack has value 11, a queen has value 12, and a king has value 13.

   (a) Compute $P(X = x)$ for every real x.
   (b) Suppose that $Y = 1, 2, 3$, or 4 when a diamond, heart, club, or spade is drawn. Compute $P(Y = y)$ for every real number y.
   (c) Compute $P(W = w)$ for every real number when w when $W = X + Y$.

(7) Suppose a university is composed of 55% female students and 45% male students. A student is selected to complete a questionnaire. There are 25 questions on the questionnaire administered to a male student and 30 questions on the questionnaire administered to a female student. If X denotes the number of questions answered by a randomly selected student, then compute $P(X = x)$ for every real number x.

(8) Suppose that a bowl contains 10 chips each uniquely numbered 0 through 9. The chips are thoroughly mixed, one is drawn and the number on it, X_1 is noted. This chip is replaced in the bowl. A second chip is drawn and the number on it, X_2 is noted. Compute $P(W = w)$ for every real number when $W = X_1 + 10X_2$.

(9) Suppose that a bowl contains 10 chips each uniquely numbered 0 through 9. The chips are thoroughly mixed, one is drawn and the number on it, X_1 is noted. This chip is *not* replaced in the bowl. A second chip is drawn and the number on it, X_2 is noted. Compute $P(W = w)$ for every real number when $W = X_1 + 10X_2$.

(10) Suppose Alice flips three fair coins, and let X be the number of heads showing. Suppose Barbara flips five fair coins, and let Y be the number of heads showing. Let $Z = X - Y$. Compute $P(Z = z)$ for every real number z.