STAT 375 Probability Theory **Quiz 9** **November 15, 2024**

Your Name:

Duration of the Quiz is 25 minutes. There are three problems, worth 20 points. Show all your work for full credit. Books, notes etc. are prohibited.

1. (a) Given $f(y) = cy^2$, $0 \leq y \leq 2$, and $f(y) = 0$ elsewhere, find the value of c for which $f(y)$ is a valid density function. Use this value for parts (b), (c) and (d).

   (b) Find $P(1 \leq Y \leq 2)$. What can you say about $P(1 < Y < 2)$?

   (c) Find $E(Y)$

   (d) Find $\text{Var}(Y)$.

2. Let Y be a continuous random variable with $f_Y(y) = ye^{-y}$, $0 \leq y$. Find the moment-generating function for Y.

3. If $M_X(t) = \left(\frac{1}{2} + \frac{1}{2}\, e^t\right)^4$ be the moment-generating function for X, find

   (a) $E(X)$

   (b) $E(X^2)$

   (c) $\mathrm{Var}(X)$