Due by 4pm on Friday, November 15. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code.

(1) (5 points each)

In Problem 1, you need to use the formula $\text{Var}(X) = E(X^2) - (E(X))^2$. To compute $\text{E}(X^2)$, sometimes you need to use the formula $E(X(X-1)) = E(X^2) - E(X)$.

- (a) Let $X \sim \text{Binomial}(n, p)$. Find $\text{Var}(X)$. Compute $E(X(X-1))$
- (b) Let $X \sim \text{Hypergeometric}(N, r, n)$, where $N = r + w$. Find $\text{Var}(X)$. Skip it
- (c) Let $X \sim \text{Geometric}(p)$. Find $\text{Var}(X)$. Compute $E(X(X-1))$
- (d) Let $X \sim \text{Negative Binomial}(p)$. Find $\text{Var}(X)$. Let X be a RV that has a Negative Binomial Distribution. Write it as a sum of r number of Geometric RVs.
- (e) Let $X \sim \text{Poissonl}(\lambda)$. Find $\text{Var}(X)$. Compute $E(X^2)$

(2) (5 points each) Find the Moment-Generating Function of a RV for a RV X that has a

- (a) Hypergeometric distribution, Skip it
- (b) Geometric Distribution, Apply the definition $E(e^{tX})$. In the summation, pull $\frac{p}{1-p}$ out, and you will see a Geometric series
- (c) Negative Binomial Distributions, and Let X be a RV that has a Negative Binomial Distribution. Write it as a sum of r number of Geometric RVs.
- (d) Poisson Distribution. Apply the definition $E(e^{tX})$. In the summation, pull $e^{-\lambda}$ out, and you will see a Taylor series of a function

(3) (10 points) Suppose that X is a random variable with moment-generating function $M_X(t)$.

- (a) What is $M_X(0)$?
- (b) If $W = aY + b$, find the moment-generating function of W. $E(e^{tW}) = E(e^{t(aY+b)}) = E(e^{tb}e^{(at)Y}) = \dots$

(4) (10 points) Let $M_X(t) = \frac{1}{6}e^t + \frac{2}{6}e^{2t} + \frac{3}{6}e^{3t}$. Find the following:

- (a) $E(X)$
- (b) $E(X^2)$
- (c) $\text{Var}(X)$
- (d) The distribution of X.

(5) (5 points) Two chips are drawn at random and without replacement from an urn that contains five chips, numbered 1 through 5. If the sum of the chips drawn is even, the random variable X equals 5; if the sum of the chips drawn is odd, $X = -3$. Find the moment-generating function for X.

(6) (5 points each)

- (a) If X has moment-generating function $M_X(t) = (.7e^t + .3)^{10}$, what is $P(X \le 5)$? Identify the mgf, and then you can find the probability of a success p
- (b) If Y has moment-generating function $M_Y(t) = e^{6(e^t - 1)}$, what is $P(|Y - \mu| \le 2\sigma)$? Identify the mgf, and then you can find λ

(7) (20 points) Let Y be an integer-valued random variable for which $P(Y = i) = p_i$, where $i = 0, 1, 2, \dots$ The probability-generating function $P(t)$ for Y is defined to be

$$P(t) = E(t^Y) = p_0 + p_1 t + p_2 t^2 + \cdots = \sum_{i=0}^{\infty} p_i t^i$$

for all values of t such that $P(t)$ is finite. Note that the coefficient of t^i in $P(t)$ is the probability p_i. Correspondingly, the coefficient of t^i in $M_X(t)$ is a constant times the ith moment μ_i.

The kth *factorial moment* for a random variable Y is defined to be

$$\mu_{[k]} = E[Y(Y-1)(Y-2)\cdots(Y-k+1)],$$

where k is an integer. Note that $\mu_{[1]} = E(Y) = \mu$, and the second factorial moment $\mu_{[2]} = E[Y(Y-1)]$ is useful in finding the variance of Y.

Prove that $P^{(1)}(1) = \mu_{[1]}$, $P^{(2)}(1) = \mu_{[2]}$, and in general, $P^{(k)}(1) = \mu_{[k]}$, and then find the probability-generating function for a geometric random variable.

$P'(t) = p_1 + 2p_2 t + 3p_3 t^2 + \cdots$

$P'(1) = p_1 + 2p_2 + 3p_3 + \cdots = \sum_{k=0}^{\infty} k \cdot p_k = \sum_{k=0}^{\infty} k \cdot P(Y = k) = E(Y)$