Due by 4pm on Friday, October 25. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code. Each problem is worth 10 points.

(1) The Stanley Cup playoff in professional hockey is a seven-game series, where the first team to win four games is declared the champion. The series, then, can last anywhere from four to seven games (just like the World Series in baseball). Calculate the likelihoods that the series will last four, five, six, or seven games. Assume that (1) each game is an independent event and (2) the two teams are evenly matched.

(2) Kingwest Pharmaceuticals is experimenting with a new affordable AIDS medication, PM-17, that may have the ability to strengthen a victim's immune system. Thirty monkeys infected with the HIV complex have been given the drug. Researchers intend to wait six weeks and then count the number of animals whose immunological responses show a marked improvement. Any inexpensive drug capable of being effective 60% of the time would be considered a major breakthrough; medications whose chances of success are 50% or less are not likely to have any commercial potential.

Yet to be finalized are guidelines for interpreting results. Kingwest hopes to avoid making either of two errors: (1) rejecting a drug that would ultimately prove to be marketable and (2) spending additional development dollars on a drug whose effectiveness, in the long run, would be 50% or less. As a tentative "decision rule," the project manager suggests that unless *sixteen or more* of the monkeys show improvement, research on PM-17 should be discontinued.

(a) What are the chances that the "sixteen or more" rule will cause the company to reject PM-17, *even if the drug is 60% effective*?

(b) How often will the "sixteen or more" rule allow a 50%-effective drug to be perceived as a major breakthrough?

(3) The junior mathematics class at Superior High School knows that the probability of making a 600 or greater on the SAT Reasoning Test in Mathematics is 0.231, while the similar probability for the Critical Reading Test is 0.191. The math students issue a challenge to their math-averse classmates. Each group will select four students and have them take the respective test. The mathematics students will win the challenge if more of their members exceed 600 on the mathematics test than do the other students on the Critical Reading Test. What is the probability that the mathematics students win the challenge?

(4) Doomsday Airlines ("Come Take the Flight of Your Life") has two dilapidated airplanes, one with two engines, and the other with four. Each plane will land safely only if at least half of its engines are working. Each engine on each aircraft operates independently and each has probability $p = 0.4$ of failing. Assuming you wish to maximize your survival probability, which plane should you fly on?

(5) If a family has four children, is it more likely they will have two boys and two girls or three of one sex and one of the other? Assume that the probability of a child being a boy is $\frac{1}{2}$ and that the births are independent events.

(6) A corporate board contains twelve members. The board decides to create a five-person Committee to Hide Corporation Debt. Suppose four members of the board are accountants. What is the probability that the Committee will contain two accountants and three non-accountants?

(7) A display case contains thirty-five gems, of which ten are real diamonds and twenty-five are fake diamonds. A burglar removes four gems at random, one at a time and without replacement. What is the probability that the last gem she steals is the second real diamond in the set of four?

(8) Urn I contains five red chips and four white chips; urn II contains four red and five white chips. Two chips are drawn simultaneously from urn I and placed into urn II. Then a single chip is drawn from urn II. What is the probability that the chip drawn from urn II is white?

(9) Is

$$p(s) = \frac{1}{1+\lambda}\left(\frac{\lambda}{1+\lambda}\right)^s,\ s = 0, 1, 2, \ldots; \quad \lambda > 0$$

a discrete probability function? Why or why not?

(10) Acme Industries typically produces three electric power generators a day; some pass the company's quality-control inspection on their first try and are ready to be shipped; others need to be retooled. The probability of a generator needing further work is 0.05. If a generator is ready to ship, the firm earns a profit of $10,000. If it needs to be retooled, it ultimately costs the firm $2,000. Let X be the random variable quantifying the company's daily profit. Find $p_X(k)$.