STAT 375 Probability Theory **Homework Assignment 4** **Fall 2024**

Due by 4pm on Friday, September 27. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code. Each problem is worth 10 points.

(1) As medical technology advances and adults become more health conscious, the demand for diagnostic screening tests inevitably increases. Looking for problems, though, when no symptoms are present can have undesirable consequences that may outweigh the intended benefits. Suppose, for example, a woman has a medical procedure performed to see whether she has a certain type of cancer. Let B denote the event that the test *says* she has cancer, and let A_1 denote the event that she actually *does* (and A_2, the event that she *does* not). Furthermore, suppose the prevalence of the disease and the precision of the diagnostic test are such that

$P(A_1) = 0.0001$ [and $P(A_2) = 0.9999$]

$P(B|A_1) = 0.90 = P(\text{Test } \textit{says} \text{ woman has cancer when, in fact, she does})$

$P(B|A_2) = P(B|A_1^c) = 0.001 = P(\text{false positive}) = P(\text{Test } \textit{says} \text{ woman has cancer when, in fact, she does not}).$

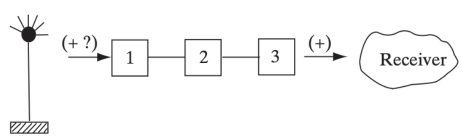
What is the probability that she *does* have cancer, given that the diagnostic procedure says she does?

(2) In the game of craps, one of the ways a player can win is by rolling (with two dice) one of the sums 4, 5, 6, 8, 9, or 10, and then rolling that sum again before rolling a sum of 7. For example, the sequence of sums 6, 5, 8, 8, 6 would result in the player winning on his fifth roll. In gambling parlance, "6" is the player's "point," and he "made his point." On the other hand, the sequence of sums 8, 4, 10, 7 would result in the player losing on his fourth roll: his point was an 8, but he rolled a sum of 7 before he rolled a second 8. What is the probability that a player wins with a point of 10?

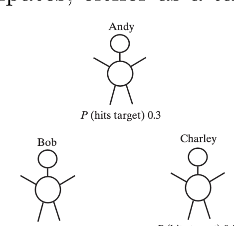
(3) A transmitter is sending a binary code (+ and − signals) that must pass a through three relay signals before being sent on to the receiver (see the following figure). At each relay station, there is a 25% chance that the signal will be reversed—that is

$$P(+ \text{ is sent by relay } i \,|\, - \text{ is received by relay } i) = P(- \text{ is sent by relay } i \,|\, + \text{ is received by relay } i) = \frac{1}{4}, \quad i = 1, 2, 3$$

Suppose + symbols make up 60% of the message being sent. If the signal + is received, what is the probability a + was sent?



(4) Andy, Bob, and Charley have gotten into a disagreement over a female acquaintance, Donna, and decide to settle their dispute with a three-cornered pistol duel. Of the three, Andy is the worst shot, hitting his target only 30% of the time. Charley, a little better, is on-target 50% of the time, while Bob never misses (see the following figure). The rules they agree to are simple: They are to fire at the targets of their choice in succession, and cyclically, in the order Andy, Bob, Charley, and so on, until only one of them is left standing. On each "turn," they get only one shot. If a combatant is hit, he no longer participates, either as a target or as a shooter.



Prove that the best strategy for Andy is to deliberately miss both Bob and Charley with the first shot, and then hit Bob on second turn.

(5) Males and females are observed to react differently to a given set of circumstances. It has been observed that 70% of the females react positively to these circumstances, whereas only 40% of males react positively. A group of 20 people, 15 female and 5 male, was subjected to these circumstances, and the subjects were asked to describe their reactions on a written questionnaire. A response picked at random from the 20 was negative. What is the probability that it was that of a male?

(6) Of the travelers arriving at a small airport, 60% fly on major airlines, 30% fly on privately owned planes, and the remainder fly on commercially owned planes not belonging to a major airline. Of those traveling on major airlines, 50% are traveling for business reasons, whereas 60% of those arriving on private planes and 90% of those arriving on other commercially owned planes are traveling for business reasons. Suppose that we randomly select one person arriving at this airport. What is the probability that the person
  (a) is traveling on business?
  (b) is traveling for business on a privately owned plane?
  (c) arrived on a privately owned plane, given that the person is traveling for business reasons?
  (d) is traveling on business, given that the person is flying on a commercially owned plane?

(7) Cards are dealt, one at a time, from a standard 52-card deck.
  (a) If the first 2 cards are both spades, what is the probability that the next 3 cards are also spades?
  (b) If the first 3 cards are all spades, what is the probability that the next 2 cards are also spades?
  (c) If the first 4 cards are all spades, what is the probability that the next card is also a spade?

(8) A survey of consumers in a particular community showed that 10% were dissatisfied with plumbing jobs done in their homes. Half the complaints dealt with plumber A, who does 40% of the plumbing jobs in the town. Find the probability that a consumer will obtain
  (a) an unsatisfactory plumbing job, given that the plumber was A.
  (b) a satisfactory plumbing job, given that the plumber was A.

(9) (a) Suppose that A and B are mutually exclusive events, with $P(A) > 0$ and $P(B) < 1$. Are A and B independent? Prove your answer.
  (b) Suppose that $A \subset B$ and that $P(A) > 0$ and $P(B) > 0$. Are A and B independent? Prove your answer.

(10) Suppose that there is a 1 in 50 chance of injury on a single skydiving attempt.
  (a) If we assume that the outcomes of different jumps are independent, what is the probability that a skydiver is injured if she jumps twice?
  (b) A friend claims if there is a 1 in 50 chance of injury on a single jump then there is a 100% chance of injury if a skydiver jumps 50 times. Is your friend correct? Why?