Due by 6pm on Wednesday, November 27. Please leave your homework on the table before class begins on Friday or leave it in the dropbox outside my office. Do not forget to attach the honor code.

(1) (10 points) Weekly CPU time used by an accounting firm has probability density function (measured in hours) given by

$$f_Y(y) = \begin{cases} (3/64)y^2(4-y), & 0 \le y \le 4, \\ 0 & elsewhere \end{cases}$$

(a) Find the expected value and variance of weekly CPU time.
(b) The CPU time costs the firm \$200 per hour. Find the expected value and variance of the weekly cost for CPU time.
(c) Would you expect the weekly cost to exceed \$600 very often? Why?

(2) (10 points) The pH of water samples from a specific lake is a random variable Y with probability density function given by

$$f_Y(y) = \begin{cases} (3/8)(7-y)^2, & 5 \le y \le 7, \\ 0 & elsewhere \end{cases}$$

(a) Find $E(Y)$ and $\mathrm{Var}(Y)$.
(b) Find an interval shorter than (5, 7) in which at least three-fourths of the pH measurements must lie.
(c) Would you expect to see a pH measurement below 5.5 very often? Why?

(3) (10 points) Wires manufactured for use in a computer system are specified to have resistances between .12 and .14 ohms. The actual measured resistances of the wires produced by company A have a normal probability distribution with mean .13 ohm and standard deviation .005 ohm.

(a) What is the probability that a randomly selected wire from company A's production will meet the specifications?
(b) If four of these wires are used in each computer system and all are selected from company A, what is the probability that all four in a randomly selected system will meet the specifications?

(4) (10 points) The magnitude of earthquakes recorded in a region of North America can be modeled as having an exponential distribution with mean 2.4, as measured on the Richter scale. Find the probability that an earthquake striking this region will
(a) exceed 3.0 on the Richter scale.
(b) fall between 2.0 and 3.0 on the Richter scale.

(5) (10 points) A gasoline wholesale distributor has bulk storage tanks that hold fixed supplies and are filled every Monday. Of interest to the wholesaler is the proportion of this supply that is sold during the week. Over many weeks of observation, the distributor found that this proportion could be modeled by a beta distribution with $\alpha = 4$ and $\beta = 2$. Find the probability that the wholesaler will sell at least 90% of her stock in a given week.

(6) (10 points) The weekly amount of downtime Y (in hours) for an industrial machine has approximately a gamma distribution with $\alpha = 3$ and $\beta = 2$. The loss L (in dollars) to the industrial operation as a result of this downtime is given by $L = 30Y + 2Y^2$. Find the expected value and variance of L.

(7) (10 points) During an eight-hour shift, the proportion of time Y that a sheet-metal stamping machine is down for maintenance or repairs has a beta distribution with $\alpha = 1$ and $\beta = 2$. That is,

$$f_Y(y) = \begin{cases} 2(1-y), & 0 \le y \le 1, \\ 0 & elsewhere \end{cases}$$

The cost (in hundreds of dollars) of this downtime, due to lost production and cost of maintenance and repair, is given by $C = 10 + 20Y + 4Y^2$. Find the mean and variance of C.

(8) (10 points) Errors in measuring the time of arrival of a wave front from an acoustic source sometimes have an approximate beta distribution. Suppose that these errors, measured in microseconds, have approximately a beta distribution with $\alpha = 1$ and $\beta = 2$.
(a) What is the probability that the measurement error in a randomly selected instance is less than .5 μs?
(b) Give the mean and standard deviation of the measurement errors.

(9) (10 points) Consider a random variable Y with density function given by

$$f_Y(y) = ke^{-y^2/2}, \ -\infty < y < \infty.$$

(a) Find k.
(b) Find the moment-generating function of Y.
(c) Find $E(Y)$ and $\mathrm{Var}(Y)$.

(10) (10 points) If $a < b$, derive the moment-generating function of a random variable that has a uniform distribution on the interval $[a, b]$,