Please do not write in the boxes immediately below.

problem	1	2	3	4	5	6	7	8	total
points									

STAT 375 Fall 2024 Exam 2

November 20, 2024

Your name____

The exam has 10 pages: front page, 9 different printed sides of exam problems and 1 side workspace.

Duration of the Midterm Exam is 90 minutes. There are 8 problems, worth 15 points each. From Problems 1 – 8, only 7 problems will be graded. If you solve all Problems 1 – 8, you must cross out the problem in the box above that must not be graded. If you solve all Problems 1 – 8 and do not cross out a problem, only the first seven problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1. A particular concentration of a chemical found in polluted water has been found to be lethal to 20% of the fish that are exposed to the concentration for 24 hours. Twenty fish are placed in a tank containing this concentration of chemical in water.

   (a) Find the probability that exactly 14 survive.

   (b) Find the probability that at least 10 survive.

   (c) Find the probability that at most 16 survive.

   (d) Find the mean and variance of the number that survive.

2. A certified public accountant (CPA) has found that nine of ten company audits contain substantial errors. If the CPA audits a series of company accounts,

   (a) what is the probability that the first account containing substantial errors is the third one to be audited?

   (b) what is the probability that the first account containing substantial errors will occur on or after the third audited account?

   (c) What are the mean and standard deviation of the number of accounts that must be examined to find the first one with substantial errors?

3. (a) In an assembly-line production of industrial robots, gearbox assemblies can be installed in one minute each if holes have been properly drilled in the boxes and in ten minutes if the holes must be re-drilled. Twenty gearboxes are in stock, 2 with improperly drilled holes. Five gearboxes must be selected from the 20 that are available for installation in the next five robots.

(i) Find the probability that all 5 gearboxes will fit properly.

(ii) Find the mean, variance, and standard deviation of the time it takes to install these 5 gearboxes.

(b) Five cards are dealt at random and without replacement from a standard deck of 52 cards. What is the probability that the hand contains all 4 aces if it is known that it contains at least 3 aces?

4. (a) Let $X \sim \text{Poisson}(\lambda)$. Find $\text{Var}(X)$ and the Moment-Generating Function for X. You must prove these results.

(b) The mean number of automobiles entering a mountain tunnel per two-minute period is one. An excessive number of cars entering the tunnel during a brief period of time produces a hazardous situation. Find the probability that the number of autos entering the tunnel during a two-minute period exceeds three.

5. (a) If X has moment-generating function $M_X(t) = [(1/3)e^t + (2/3)]^5$, what is $P(X \leq 3)$?

(b) If X has moment-generating function $M_X(t) = e^{2(e^t - 1)}$, what is $P(1.1 \leq X \leq 10.1)$?

(c) If $M_X(t) = \dfrac{e^t}{2 - e^t}$ be the moment-generating function for X, find

i. $E(X)$

ii. $E(X^2)$

iii. $\text{Var}(X)$

6. (i) If Z is a standard normal random variable, what is

(a) $P(0 \leq Z \leq 2.07)$

(c) $P(Z \geq 4.61)$

(b) $P(Z < -2.33)$

(d) $P(0 \leq Z \leq 2.07)$

(iii) Evaluate $\int_{-0.44}^{1.33} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz.$

(iv) For what value of z is the following statement true? $P(-1.00 \leq Z \leq z) = 0.5004$.

(ii) A company that manufactures and bottles apple juice uses a machine that automatically fills 16-ounce bottles. There is some variation, however, in the amounts of liquid dispensed into the bottles that are filled. The amount dispensed has been observed to be approximately normally distributed with mean 16 ounces and standard deviation 1 ounce. Determine the probability of bottles that will have more than 17 ounces dispensed into them.

7. The proportion of time Y that an industrial robot is in operation during a 40-hour week is a random variable with probability density function

$$f_Y(y) = \begin{cases} 2y, & 0 \le y \le 1, \\ 0 & elsewhere \end{cases}$$

(a) Find $E(Y)$ and $\mathrm{Var}(Y)$.

(b) For the robot under study, the profit X for a week is given by $X = 200Y - 60$. Find $E(X)$ and $\mathrm{Var}(X)$.

(c) Find an interval in which the profit should lie for at least 75% of the weeks that the robot is in use.

8. The length of time required by students to complete a one-hour exam is a random variable with a density function given by

$$f_Y(y) = \begin{cases} cy^2 + y, & 0 \leq y \leq 1, \\ 0 & elsewhere \end{cases}$$

(a) Find c.

(b) Find $F_Y(y)$

(c) Graph $f_Y(y)$ and $F_Y(y)$.

(d) Use $F_Y(y)$ in part (b) to find $F(-1)$, $F(0)$, and $F(1)$.

(e) Find the probability that a randomly selected student will finish in less than half an hour.

WORKSHEET