Please do not write in the boxes immediately below.

problem	1	2	3	4	5	6	EC	total
points								

STAT 375 Fall 2024 Exam 1

October 9, 2024

Your name____

The exam has 9 different printed sides of exam problems and 1 side workspace.

Duration of the Midterm Exam is 90 minutes. There are 6 problems, worth 20 points each, and an extra credit problem, worth 10 points. From Problems 1 – 6, only 5 problems will be graded. If you solve all Problems 1 – 6, you must cross out the problem in the box above that must not be graded. If you solve all Problems 1 – 6 and do not cross out a problem, only the first five problems will be graded. Show all your work for full credit. Books, notes etc. are prohibited. Calculators, cellphones, earphones, AirPods and cheat sheets are NOT permitted.

1. There are two parts in this problem. Please go to next page for Part (b).

   (a) Let S be a sample space, and A and B be events defined on S. Prove the following. You must mention any result, theorem or axiom used.

   (i) $P(\emptyset) = 0$

   (ii) If $A \subseteq B$, then $P(A) \leq P(B)$.

(b) Suppose that A and B are two events such that $P(A) = .6$ and $P(B) = .3$.

(i) Is it possible that $P(A \cap B) = .1$? Why or why not?

(ii) What is the smallest possible value for $P(A \cap B)$?

(iii) Is it possible that $P(A \cap B) = .7$? Why or why not?

(iv) What is the largest possible value for $P(A \cap B)$?

2. (a) Suppose we roll two fair six-sided dice, one red and one blue. Let A be the event that the two dice show the same value. Let B be the event that the sum of the two dice is equal to 12. Let C be the event that the red die shows 4. Let D be the event that the blue die shows 4.

   (i) Are A and B independent?

   (ii) Are A and C independent?

   (iii) Are C and D independent?

   (b) Suppose we roll three fair six-sided dice. Compute the conditional probability that the first die shows 4, given that the sum of the three numbers showing is 12.

3. (a) Suppose that, in a particular city, airport A handles 50% of all airline traffic, and airports B and C handle 30% and 20%, respectively. The detection rates for weapons at the three airports are 0.9, 0.5, and 0.4, respectively. If a passenger at one of the airports is found to be carrying a weapon through the boarding gate, what is the probability that the passenger is using airport A? Airport C?

(b) Suppose urn 1 has 3 red and 2 blue balls, and urn 2 has 4 red and 7 blue balls. Suppose one of the two urns is selected, and then one of the balls within that urn is picked uniformly at random. What is the probability that the ball was picked from urn 1 given that the ball was blue?

4. (a) How many integers between 1100 and 9999 have distinct digits, and how many of those are even numbers?

(b) In how many ways can the letters in MASSACHUSETTS be arranged so that

(i) no two S's are adjacent?

(ii) all S's are adjacent?

(c) What is the coefficient of x^{28} in the expansion of $(1 + x^4 + x^7)^{100}$?

(d) A boat has a crew of eight: Two of those eight can row only on the stroke side, while three can row only on the bow side. In how many ways can the two sides of the boat be manned?

5. (a) Six dice are rolled one time. What is the probability that each of the six faces appears?

(b) An urn contains six chips, numbered 1 through 6. Two are chosen at random and their numbers are added together. What is the probability that the resulting sum is equal to 5?

(c) A bridge hand (thirteen cards) is dealt from a standard 52-card deck. Let A be the event that the hand contains four aces; let B be the event that the hand contains four kings. Find $P(A \cup B)$.

6. (a) A four-sided die, numbered 1 through 4, is rolled twice. Define X to be the larger of the two outcomes if they are different and the common value if they are the same. Find $P(X = x)$ for every real number x. Can you find a formula for $P(X = x)$ in terms of x?

(b) Suppose there are 3 defective items in a lot (collection) of 50 items. A sample of size 10 is taken at random and without replacement. Let X denote the number of defective items in the sample. Find

(i) $P(X \leq 1)$

(ii) $P(X = x)$ for every real number x.

Extra Credit Problem 1 (5 points) Evaluate the sum $\sum_{k=0}^{n} (-1)^k \binom{n}{k} 10^k$

Extra Credit Problem 2 (5 points) Consider a set of ten urns, nine of which contain three white chips and three red chips each. The tenth contains five white chips and one red chip. An urn is picked at random. Then a sample of size 3 is drawn without replacement from that urn. If all three chips drawn are white, what is the probability that the urn being sampled is the one with five white chips?

WORKSHEET