Counting Permutations (when the objects are all distinct)

Theorem The number of permutations of length k that can be formed from a set of n distinct elements, repetitions not allowed, is denoted by the symbol ${}_nP_k$, where

$$ {}_nP_k = n(n-1)(n-2)\cdots(n-k+1) = \frac{n!}{(n-k)!} $$

Corollay The number of ways to permute an entire set of n distinct objects is

$$ {}_nP_n = n(n-1)(n-2)\cdots 1 = n!. $$

Example 1 How many permutations of length $k = 3$ can be formed from the set of $n = 4$ distinct elements, A, B, C, and D?

Example 2 Consider the set of nine-digit numbers that can be formed by rearranging without repetition the integers 1 through 9. For how many of those permutations will the 1 and the 2 precede the 3 and the 4? That is, we want to count sequences like 7 2 5 1 3 6 9 4 8 but not like 6 8 1 5 4 2 7 3 9.

Example 3 The board of a large corporation has six members willing to be nominated for office. How many different "president/vice president/treasurer" slates could be submitted to the stockholders?

Example 4 A three-digit number is to be formed from the dig- its 1 through 7, with no digit being used more than once. How many such numbers would be less than 289?

Example 5 In how many ways can a pack of fifty-two cards be dealt to thirteen players, four to each, so that every player has one card of each suit?

Counting Permutations (when the objects are not all distinct)

Theorem The number of ways to arrange n objects, n_1 being of one kind, n_2 of a second kind, $\cdots$, and n_r of an rth kind, is

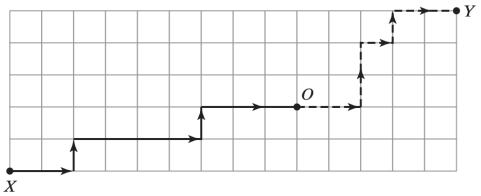
$$ \frac{n!}{n_1!n_2!\cdots n_r!} $$

where $\sum_{i=1}^{r} n_i = n$.

Example 1 A pastry in a vending machine costs 85 cents. In how many ways can a customer put in two quarters, three dimes, and one nickel?

Order in which coins are deposited

Example 2 A deliveryman is currently at Point X and needs to stop at Point 0 before driving through to Point Y (see the following figure). How many different routes can he take without ever going out of his way?



Example 3 How many numbers greater than four million can be formed from the digits 2, 3, 4, 4, 5, 5, 5?

Example 4 What is the coefficient of x^{12} in the expansion of $(1 + x^3 + x^6)^{18}$?

Example 5 What is the coefficient of $w^3x^2y^2z^2$ in the expansion of $(w + x + y + z)^9$?

Example 6 Which state name can generate more permutations, MISSISSIPPI or GEORGIA?