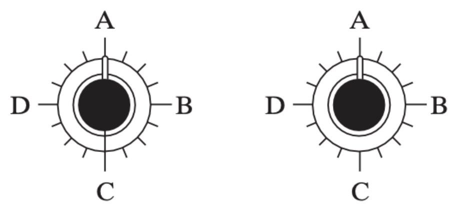
## Counting Ordered Sequences: The Multiplication Rule

**Multiplication Rule** If operation $A$ can be performed in $m$ different ways and operation $B$ in n different ways, the sequence (operation $A$, operation $B$) can be performed in $m \cdot n$ different ways.

**Corrolary** If operation $A_i$, $i = 1, 2, ..., k$, can be performed in $n_i$ ways, $i = 1, 2, ..., k$, respectively, then the ordered sequence (operation $A_1$, operation $A_2$, . . ., operation $A_k$) can be performed in $n_1 \cdot n_2 \cdots n_k$ ways.

**Example 1** The combination lock on a briefcase has two dials, each marked off with sixteen notches (see the following figure). To open the case, a person first turns the left dial in a certain direction for two revolutions and then stops on a particular mark. The right dial is set in a similar fashion, after having been turned in a certain direction for two revolutions. How many different settings are possible?




**Example 2** Alphonse Bertillon, a nineteenth-century French criminologist, developed an identification system based on eleven anatomical variables (height, head width, ear length, etc.) that presumably remain essentially unchanged during an individual's adult life. The range of each variable was divided into three subintervals: small, medium, and large. A person's Bertillon configuration is an ordered sequence of eleven letters, say,

$$s, s, m, m, l, s, l, s, s, m, s$$

where a letter indicates the individual's "size" relative to a particular variable. How populated does a city have to be before it can be guaranteed that at least two citizens will have the same Bertillon configuration?

**Example 3** The annual NCAA ("March Madness") basketball tournament starts with a field of sixty-four teams. After six rounds of play, the squad that remains unbeaten is declared the national champion. How many different configurations of winners and losers are possible, starting with the first round? Assume that the initial pairing of the sixty-four invited teams into thirty-two first-round matches has already been done.

**Example 4** How many terms will be included in the expansion of

$$(a + b + c)(d + e + f)(x + y + u + v + w)$$

Which of the following will be included in that number: $aeu, cdx, bef, xvw$?

**Example 5** Suppose that the format for license plates in a certain state is two letters followed by four numbers.

(1) How many different plates can be made?
(2) How many different plates are there if the letters can be repeated but no two numbers can be the same?
(3) How many different plates can be made if repetitions of numbers and letters are allowed except that no plate can have four zeros?

**Example 6** How many integers between 100 and 999 have distinct digits, and how many of those are odd numbers?

**Example 7** Suppose that two cards are drawn—in order— from a standard 52-card poker deck. In how many ways can the first card be a club and the second card be an ace?

**Example 8** When they were first introduced, postal zip codes were five-digit numbers, theoretically ranging from 00000 to 99999. (In reality, the lowest zip code was 00601 for San Juan, Puerto Rico; the highest was 99950 for Ketchikan, Alaska.) An additional four digits have been added, so each zip code is now a nine-digit number. How many zip codes are at least as large as 60000–0000, are even numbers, and have a 7 as their third digit?