MATH 241

Worksheet 6

- 1. Let S be the solid region which is inside the sphere given by $x^2 + y^2 + z^2 = 4$ and above the plane where z = 1. Assume that x, y, and z are measured in meters. Suppose the density of the solid region S is given by $\delta(x, y, z) = \frac{1}{x^2 + y^2 + z^2} \text{ kg/m}^3$. Find the total mass of the solid region S.
- 2. Use spherical coordinates to find the volume of the solid that lies above the cone $z = \sqrt{x^2 + y^2}$ and below the sphere $x^2 + y^2 + z^2 = z$.
- 3. Let S be the solid region in the 1st octant (i.e., where $x \ge 0$, $y \ge 0$, and $z \ge 0$) in \mathbb{R}^3 which lies inside the (top) hemisphere where $x^2 + y^2 + z^2 = 4$ and $z \ge 0$ but lies outside the cylinder where $x^2 + y^2 = 1$, and bounded by the planes with equations y = x and $y\sqrt{3} = x$. Compute the volume of the solid region S.
- 4. Let S be the solid region which is bounded by the half-cones where $z = \sqrt{x^2 + y^2}$ and $z\sqrt{3} = \sqrt{x^2 + y^2}$, and inside the cylinder where $x^2 + y^2 = 4$ and outside the cylinder where $x^2 + y^2 = 1$. Assume that x, y, and z are measured in meters. Suppose the density of the solid region S is given by $\delta(x, y, z) = \frac{1}{x^2 + y^2 + z^2} \text{ kg/m}^3$. Find the total mass of the solid region S.
- 5. Find the volume of the region in the following figure.



- 6. Let S be the solid region above the sphere $x^2 + y^2 + z^2 = 6$ and below the paraboloid $z = 4 x^2 y^2$.
 - (a) Sketch the projection of S onto the xy-plane.
 - (b) Compute the volume of the solid S.



- 7. Calculate the triple integral of f(x, y, z) over the given region.
 - (a) f(x, y, z) = 1; $x^2 + y^2 + z^2 \le 4z$; $z \ge \sqrt{x^2 + y^2}$ (b) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$; $x^2 + y^2 + z^2 \le 2z$
- 8. Express the triple integral in cylindrical coordinates.

$$\int_0^2 \int_0^{\sqrt{2x-x^2}} \int_0^{\sqrt{x^2+y^2}} f(x,y,z) \, dz \, dy \, dx$$