### Section 13.2 - Line Integrals

1. Consider the force field $\mathbf{F}(x, y, z) = (xz, 0, -yz)$ in Newtons, where $x$, $y$, and $z$ are in meters. Let $C$ be the oriented line segment from $(-1, 2, 0)$ to $(3, 0, 1)$. Calculate the work done by $\mathbf{F}$ along $C$.

2. Consider the force field $\mathbf{F}(x, y) = (xy, -x + y)$ in Newtons, where $x$ and $y$ are in meters. Let $C_1$ be the oriented curve from $(0, 0)$ to $(1, 1)$ along $y = x$. Let $C_2$ be the oriented curve from $(0, 0)$ to $(1, 1)$ along $y = x^2$. Calculate the work done by $\mathbf{F}$ along $C_1$ and $C_2$.

3. Let $\mathbf{F} = (x^2 + y^2, xy)$. Let $C$ be the oriented upper semi circle of radius 3 from $(3, 0)$ to the point $(-3, 0)$. Find the work done by $\mathbf{F}$ along $C$.

### Section 13.3 - Conservative Vector Fields

4. Consider the vector field $\mathbf{F} = (2x^3y^4 + x, 2x^4y^3 + y)$.

   a) Show that $\mathbf{F}$ is conservative.

   b) Calculate the line integral $\int_C \mathbf{F} \cdot \mathrm{d}r$, where $C$ is the oriented piece of the parabola given by $y = x^2$ from $(0, 0)$ to $(1, 1)$.

5. Consider the vector field $\mathbf{F} = (2y + 8xy^3, 2x + 12x^2y^2)$. Find the work done by $\mathbf{F}$ along $C = C_1 + C_2 + C_3$ where

   $C_1$: oriented line segment from $(2, 0)$ to $(2, 3)$,

   $C_2$: oriented line segment from $(2, 3)$ to $(0, 3)$,

   $C_3$: oriented quarter of the circle of radius 3, centered at the origin, from $(0, 3)$ to $(-3, 0)$.

6. Consider the vector field $\mathbf{F} = (4x - z, 3y + z, y - x)$.

   a) Show that $\mathbf{F}$ is conservative.

   b) Calculate the line integral $\int_C \mathbf{F} \cdot \mathrm{d}r$, where $C$ is an oriented curve from $(1, 2, 3)$ to $(2, 0, 1)$.

7. Let $f(x, y, z) = x^2 + y^3 + z^4$ and $\mathbf{F} = \vec{\nabla} f$. Find the line integral of $\mathbf{F}$ along the oriented curve, consisting of four line segments, which go from $(1, 0, 0)$ to $(1, 2, 5)$, then from $(1, 2, 5)$ to $(2, -3, 7)$ , then from $(2, -3, 7)$ to $(-4, 6, -7)$, and then from $(-4, 6, -7)$ to $(0, 0, 1)$.

### Section 13.4 - Green's Theorem

8. Let $\mathbf{F} = (x, -x^2y^2)$ be a force field in Newtons. Calculate the work done by $\mathbf{F}$ on an object that starts at (0,0), travels along a line to (1,1), then travels along a line to (0,1), and finally travels along a line back to (0,0).

9. Let $\mathbf{F} = (x^5 - y^3, x^3 - y^5)$, and $C$ be the curve which starts at (0,0), moves along a line segment to $(1/\sqrt{2}, 1/\sqrt{2})$, moves counterclockwise along the circle of radius 1, centered at the origin, to the point (0,1), and then moves long a line segment, back to (0,0). Calculate the work done by $F$ along $C$.

10. Let $\mathbf{F} = (x^5 - y^3, x^3 - y^5)$, and $C_1$ be the curve which starts at (0,0), moves along a line segment to $(1/\sqrt{2}, 1/\sqrt{2})$, moves counterclockwise along the circle of radius 1, centered at the origin, to the point (0,1). Find the line integral along $C_1$.

11. Calculate $\int_C \mathbf{F} \cdot \mathrm{d}r$, where $\mathbf{F} = (y^2 + e^x,\ 2xy + 5x + e^y)$ and $C$ is the circle of radius 5 centered at $(1, 1)$ and oriented clockwise.