Cylindrical Coordinates and Spherical Coordinates

1. Evaluate $\iiint_S 16z\, dV$, where S is the solid upper hemisphere of the sphere $x^2 + y^2 + z^2 = 4$.

2. Evaluate $\iiint_E y\ dV$ where E is the region that lies below the plane $z = x + 2$ above the xy-plane and between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

3. Let S be the solid region bounded by $z = \sqrt{3(x^2 + y^2)}$ and above by $x^2 + y^2 + z^2 = 4$. Find the volume of the solid region S.

4. Let S be the solid region inside the sphere given by $x^2 + y^2 + z^2 = 4$ and outside the right circular cylinder given by $x^2 + y^2 = 1$.

   (a) Set up an integral in cylindrical coordinates to compute the volume of the solid region S.

   (b) Set up an integral in spherical coordinates to compute the volume of the solid region S.

5. Find the volume of S, using whatever coordinates seem to be the most convenient. S is the solid region outside the cylinder where $x^2 + y^2 = 1$, inside the cylinder where $x^2 + y^2 = 4$, and inside the sphere of radius 3, centered at the origin.

6. Find the volume of the solid region between the spheres of radius 3 and radius 5, centered at the origin, and inside the cone where $z = \sqrt{x^2 + y^2}$.

7. Let S be the solid region in the 1st octant (i.e., where $x \geq 0$, $y \geq 0$, and $z \geq 0$) in $\mathbb{R}^3$ which is contained within the sphere where $x^2 + y^2 + z^2 = 16$, bounded by the cones where $z = \sqrt{x^2 + y^2}$ and $z\sqrt{3} = \sqrt{x^2 + y^2}$, and bounded by the planes with equations $y = x$ and $y\sqrt{3} = x$. Find the volume of S.

8. Evaluate $\displaystyle\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} z\, dz\, dx\, dy$

Density and Mass

9. Let S be the solid region in the first quadrant inside the sphere of radius 2, and outside the sphere of radius 1, both centered at the origin. Find the mass of S if the density of the solid region S is given by $\delta(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ kg/m^3.

10. Let S be the solid right circular cylinder where $-4 \leq z \leq 4$ and $0 \leq x^2 + y^2 \leq 4$. Find the mass of S if the density of the solid region S is given by $\delta(x, y, z) = \sqrt{x^2 + y^2}$ kg/m^3.

11. Let S be the solid region under the graph of $z = 9 - x^2 - y^2$ and above the plane where $z = 5$. Find the mass of S if the density of the solid region S is given by $\delta(x, y, z) = kz$ kg/m^3, where $k > 0$.

12. Let S be the solid circular half-cylinder where $-4 \leq z \leq 4$, $0 \leq x^2 + y^2 \leq 4$, and $y \geq 0$. Find the mass of S if the density of the solid region S is given by $\delta(x, y, z) = y$ kg/m^3.

13. Let S be the solid region above the rectangle, in the xy-plane, where $0 \leq x \leq 1$ and $0 \leq y \leq 5$, below the hyperbolic paraboloid given by $z = 2 + y^2 - x^2$, and above the plane where $z = 1$. Find the mass of S if the density of the solid region S is given by $\delta(x, y, z) = z$ kg/m^3.

14. Assume that a region S in $\mathbb{R}^3$ is occupied by a solid object and that, at each point (x, y, z) in the object, you have the density $\delta(x, y, z)$. Show that the mass of the object is the volume of S times the average value of the density function on the region S.