Section 12.7 - Triple Integrals

1. (a) Evaluate $\iiint_S x\,dV$, where S is the solid region under the plane $2x+3y+z=6$ that lies in the first octant.
   (b) Find the volume of the solid region S in part (a)

2. Let S be the solid region which is bounded on the sides and top by the planes where $x=0$ and $x+z=1$ and on the bottom by the parabolic cylinder where $z=y^2-1$.

   (a) Sketch the solid region S.

   (b) Sketch the projection of the solid region S onto the xy-plane and set up an iterated integral to compute the volume of the solid region S.

   (c) Sketch the projection of the solid region S onto the yz-plane and set up an iterated integral to compute the volume of the solid region S.

   (d) Sketch the projection of the solid region S onto the xz-plane and set up an iterated integral to compute the volume of the solid region S.

3. Integrate $f(x,y,z)=x$ over the solid region W bounded above by $z=4-x^2-y^2$ and below by $z=x^2+3y^2$ in the first octant $x\geq 0, y\geq 0, z\geq 0$.

4. Find the volume of the solid region S between the planes $z=x+y$ and $z=3x+5y$ lying over the rectangle $D=[0,3]\times[0,2]$.

5. Find the volume of the solid region S between the planes $z=x+y$ and $z=3x+5y$ lying over the triangle with vertices $(0,0)$, $(1,0)$ and $(0,1)$.

6. (a) Consider the integral $\iiint_S x\,dV$ over the solid region S which is under the graph of $z=4-x^2-y^2$ and above the first quadrant in the xy-plane. Write down the integral with limits.
   (b)Write down the integral to find the volume of the solid region S in part (a).