Directional Derivatives

1. Let $f(x, y) = ye^x + y^3 + \sin x$ and suppose we are at the point $(0, 1)$.

   (a) If we move towards the point $(3, 2)$, what is the rate of change of f with respect to distance?

   (b) In what direction should we head to maximize the rate of change of f, and what is the rate of change if we move in that direction?

   (c) In what direction should we head to minimize the rate of change of f, and what is the rate of change if we move in that direction?

2. Suppose that the temperature measured in degrees Celsius at each point of a metal plate is given by $T(x, y) = e^x \cos y + e^y \cos x$, where x and y are given in meters.
(a) In what direction does the temperature increase most rapidly at the point (0,0)?

(b) What is the rate of increase in that direction?

(c) In what direction does the temperature decrease most rapidly at the point (0,0)?

3. The air temperature in a room at a point (x, y, z) is given by

$$T(x, y, z) = z(x - 1)^2 + y^3 \quad ^\circ\text{C},$$

where x, y, z are measured in meters. In which direction (as a unit vector in $\mathbb{R}^3$) should a bug at the point $(2, 1, 1)$ fly in order to cool off the fastest, and at what rate (in degrees Celsius per meter) will the air temperature around it change if it moves in that direction?

4. Suppose that the xy-plane has an electric charge, in coulombs, given by

$$Q = xy + x^2 \sin(\pi y),$$

where x and y are in meters. Also, suppose that a particle is at the point $(1, 1)$.

(a) If the particle moves in the direction in which the charge increases most rapidly, in what direction does it move?

(b) If the particle moves in the direction given in part (a), with speed $1\,\text{m/s}$, at what rate is the charge changing, in coulombs per second, when the particle is at $(1, 1)$?

(c) If the particle moves in the direction given in part (a), with speed $7\,\text{m/s}$, at what rate is the charge changing, in coulombs per second, when the particle is at $(1, 1)$?