The Chain Rule

1. Suppose that $T(u, v, w) = 3u^2 + v^2 - 2w^2$, and that $u = r\cos t$, $v = 5r - 3t$, $w = r^2 e^t$.

   (a) What is (u, v, w) when $(r, t) = (2, 0)$?

   (b) Calculate $\dfrac{\partial T}{\partial r}$ and $\dfrac{\partial T}{\partial t}$ at the point $(r, t) = (2, 0)$.

2. The pressure P, in atmospheres (atm), produced by oxygen in a bottle, with a piston, is given by

$$P = \frac{nRT}{V - 0.03n} - 1.4\left(\frac{n}{V}\right)^2,$$

where n is the number of moles of gas, T is the temperature in Kelvins (K), V is the volume in liters, and R is the gas constant 0.082 L-atm/mol-K. (Note that the constants 0.03 and 1.4 in the formula are assumed to have the appropriate units.)

a) Find $\frac{\partial P}{\partial V}$, $\frac{\partial P}{\partial T}$.

b) Suppose that n is held a constant at $n = 5$. Also, suppose, when $V = 5$ liters and $T = 300$ K, that V is increasing at a rate of 0.5 liters/s and T is increasing at a rate of 10 K/s. Find the rate of change of P, with respect to time, at this moment.

3. Suppose that the temperature in $^\circ$ C of a heated metal plate is given by $T(x,y)$, where x and y are measured in meters. Suppose that we know that

$$\nabla T(1,2) = (3,-5)\ ^\circ\text{C/m}, \quad \nabla T(2,3) = (5,-3)\ ^\circ\text{C/m}, \quad \text{and} \quad \nabla T(-3,4) = (-3,1)\ ^\circ\text{C/m}.$$

The position on the plate of an ant, at time t seconds, is given by $\mathbf{p}(t) = \langle 2t-5, 3t^2+1\rangle$ meters. What is the rate of change of the temperature at the ant's location, in $^\circ$C per second, at time $t = 1$ second?

4. Suppose that $g(x, y)$ is a differentiable real-valued function such that

$$\left.\frac{\partial g}{\partial x}\right|_{(5,1)} = 2, \qquad \left.\frac{\partial g}{\partial y}\right|_{(5,1)} = 3, \qquad \left.\frac{\partial g}{\partial x}\right|_{(2,3)} = 5, \qquad \left.\frac{\partial g}{\partial y}\right|_{(2,3)} = -4.$$

Suppose that $x = uv + v^2$ and $y = u^3 - uv^2$. Calculate $\dfrac{\partial g}{\partial v}$ when $(u, v) = (-1, 2)$.