Section 13.6 - Flux through a Surface

1. Let $\mathbf{V}(x,y,z) = (ze^y,\, z\tan^{-1}x,\, x^2+y^2+z^2)$, and $\mathbf{r}(u,v) = (u\cos v,\, u\sin v,\, 0)$, where $0 \le u \le 2$ and $0 \le v \le 2\pi$. If M is the image of the parameterization $\mathbf{r}$, oriented upward, calculate the flux through M.
2. Let $\mathbf{V}(x,y,z) = (x, y, 2x+2y)$, and M be the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the unit disk, centered at the origin in the xy-plane, with outward orientation. Calculate the flux integral of $\mathbf{V}$ through M.
3. Let $\mathbf{V}(x,y,z) = (x,y,z)$, and M be the disk of radius 5, centered at (0,7,0), in the plane where $y = 7$, oriented so that the positive direction is in the direction of the positive y-axis. Calculate the flux integral of $\mathbf{V}$ through M.

Section 13.8 - The Divergence Theorem

4. Let $\mathbf{V} = (3x, 2y, x+y)$. Let E be the solid region where $x^2+y^2+z^2 \le 9$ and $z \ge 0$. Let M be the boundary of E, oriented outward. Find the flux of $\mathbf{V}$ through M.
5. Let $\mathbf{V} = (3x, 2y, x+y)$. Let M be the top hemisphere of radius 3, centered at the origin, oriented outward. Find the flux of $\mathbf{V}$ through M.
6. Let $\mathbf{F}(x,y,z) = (x,y,z)$, and M consists of the portion of the graph of $z = 4 - x^2 - y^2$, where $z \ge 0$, oriented upward. Find the flux of $\mathbf{F}$ through M.
7. Let $\mathbf{V}(x,y,z) = (-y, x, z)$, and M be the right circular cylinder with no top and bottom, centered around the z-axis, of radius 5, between $z = -5$ and $z = 5$, oriented outward. Find the flux of $\mathbf{V}$ through M.
8. Let $\mathbf{V}(x,y,z) = (x, y, 2x+2y)$, and M be the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the unit disk, centered at the origin in the xy-plane, with outward orientation. Calculate the flux integral of $\mathbf{V}$ through M.

Section 13.7 - Stokes' Theorem

9. Let $\mathbf{F} = (z + \sin x,\, x + e^y,\, y + z^3)$. Let C be the circle, in the plane $y = z$, of radius 3, centered at the origin. Let M be the closed disk bounded by C, so that $C = \partial M$. We orient M with the normal vector with the positive z coordinate. Calculate the line integral $\int_{\partial M} \mathbf{F} \cdot d\mathbf{r}$.
10. Let $\mathbf{F} = (e^x + y,\, x + \sin y + 2z,\, 3x - 5y + z^2)$. Let M_1 be the upper-hemisphere, of radius 1, centered at the origin. Let M_2 be the portion of the circular paraboloid where $z = 1 - x^2 - y^2$, and $z \ge 0$. Both surfaces are oriented upward. Calculate the flux integrals of the curl of $\mathbf{F}$ over M_1 and M_2.
11. Consider the vector field $\mathbf{F}(x,\, y,\, z) = (2y\cos z\,,\, e^x\sin z\,,\, e^z)$. Let M be the **top hemisphere** of the sphere of radius 3, centered at the origin, oriented upward. Compute the flux integral $\displaystyle\int\int_M (\overrightarrow{\nabla} \times \mathbf{F}) \cdot \mathbf{n}\, dS$.
12. Let $\mathbf{F} = (x^3 + 2y,\, \sin y + z,\, x + \sin(z^2))$. Let C be the curve that is the intersection of the plane $z = x + 4$ with the cylinder $x^2 + y^2 = 4$, oriented counterclockwise as viewed from above. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$.