MATH 241 - 01 Multivariable Calculus **Midterm Exam 2** **November 17, 2022**

Your Name:

Duration of the exam is 90 minutes. There are seven problems, worth 50 points. Show all your work for full credit. Books, notes etc. are prohibited. Calculators are NOT permitted.

1. (a) (3 pts) Suppose that the tangent plane to $z = f(x, y)$ at $(-2, 3, 4)$ has equation $4x + 2y + z = 2$. Estimate $f(-2.1, 3.1)$.

(b) (4 pts) Find the points on the graph of $z = xy^3 + 8y^{-1}$ where the tangent plane is parallel to $2x + 7y + 2z = 0$.

2. (a) (4 pts) Gravitational and electromagnetic forces are acting on a charged particle in space. Let $F = F(x, y, z)$ be the magnitude of the net force, in Newtons, which acts on the particle at the position (x, y, z) meters. Suppose that, at $(x, y, z) = (1, 3, 2)$, the partial derivatives of F , in N/m, are

$$F_x = 4.5, \quad F_y = 10, \quad \text{and} \quad F_z = -2$$

If the position of the particle, in meters, at time t seconds, is given by

$$\mathbf{p}(t) = \langle t^2, t^3 + 2, t + 1 \rangle$$

what is the rate of change of the magnitude of the net force on the particle, with respect to time, at time $t = 1$?

(b) (4 pts) Suppose that the elevation z of a hill is given in terms of x and y coordinates (at sea level, approximating the Earth as being flat) by

$$z = 400 - 5x^2 - 3y^2$$

where x, y, and z are in feet, and $5x^2 + 3y^2 \leq 400$. Note that directions in the xy-plane would correspond to what you would read on a compass; this is to be contrasted with directions in xyz-space.

If we're at the point on the surface of the hill where $(x, y) = (2, 10)$, in what xy-direction should we head to ascend the hill as rapidly as possible?

3. (6 pts) Find the critical points of the following function, and classify each one as a point where f has a local maximum value, a local minimum value, or a saddle point.

$$f(x, y) = 4xy - x^4 - y^4$$

4. (a) (6 pts) A metal plate occupies the square region where $1 \leq x \leq 2$ and $1 \leq y \leq 2$, with x and y measured in meters. Suppose that the electric potential at each point on the plate, produced by point-charges at $(0,0)$ and $(3,3)$ is

$$P(x,y) = -\frac{1}{\sqrt{x^2+y^2}} - \frac{2.25}{\sqrt{(x-3)^2+(y-3)^2}} \text{ joules.}$$

Does the electric potential attain the global extreme values inside the region or on the boundary? Justify your answer and mention any theorem used.

(b) (1 pt each) State whether the following statements are true or false.

(i) The functions $f(x,y)$ and $g(x,y) = f(x,y) + 2002$ do not have the same critical points.

(ii) A function $f(x,y)$, defined on a closed region D in $\mathbb{R}^2$, for which the absolute minimum and the absolute maximum are the same must be constant.

(iii) The functions $f(x,y)$ and $g(x,y) = (f(x,y))^2$ have the same critical points.

(iv) If a function $f(x,y) = ax + by$ has a critical point, then $f(x,y) = 0$ for all (x,y).

(v) If (x_0, y_0) is the maximum of $f(x,y)$ on the disk $x^2 + y^2 \leq 1$, then $x_0^2 + y_0^2 < 1$.

5. (a) (5 pts) For the following sum of integrals $\int_0^1 \int_0^{x^2} y\,dy\,dx + \int_1^2 \int_0^{2-x} y\,dy\,dx$, reverse the order of integration and evaluate the resulting integral using the new order.

(b) (4 pts) Suppose that a thin plate occupies the triangular region R in the xy-plane which has vertices $(0,0)$, $(1,0)$, and $(0,1)$, where all distances are in feet. Suppose that the area-density of the plate, in slugs/ft^2, is given by

$$\delta_{\text{ar}}(x,y) = 1 + 2x + 4y$$

Find the mass of the plate.

6. (3 pts) Use polar coordinates to combine the sum

$$\int_{1/\sqrt{2}}^{1}\int_{\sqrt{1-x^2}}^{x} xy\,dy\,dx + \int_{1}^{\sqrt{2}}\int_{0}^{x} xy\,dy\,dx + \int_{\sqrt{2}}^{2}\int_{0}^{\sqrt{4-x^2}} xy\,dy\,dx$$

into one double integral. Then evaluate the double integral.

7. (2 pts each)

(a) Let S be the solid tetrahedron in the 1st octant, bounded by the three coordinate planes and the plane where $x+2y+z=3$. Sketch the projection of the solid region S onto the xz-plane and **set up** an iterated integral to compute the volume of the solid region S.

(b) Let S be the solid which is below the graph of $z = 1 - x^2 - y^2$ and above the triangle with vertices $(0,0,0)$, $(1,0,0)$, and $(0,1,0)$. **Set up** an iterated integral to compute the volume of the solid region S.

(c) Let S be the solid in $\mathbb{R}^3$ bounded by $y = x^2$, $x = y^2$, $z = x + y + 5$, and $z = 0$. **Set up** an iterated integral to compute the volume of the solid region S.