Your Name:

Duration of the exam is 90 minutes. There are six problems, worth 50 points, and an extra credit problem, worth 2 points. Show all your work for full credit. Books, notes etc. are prohibited. Calculators are NOT permitted.

1. (i) (3 pts) Find a vector equation, parametric equations and symmetric equations for the line that passes through $(1, 1, 1)$ and is parallel to the line through $(2, 0, -1)$ and $(4, 1, 3)$.

   (ii) (4 pts) Determine whether the lines $\mathbf{r}_1(t) = \langle 0, 1, 1\rangle + t\langle 1, 1, 2\rangle$ and $\mathbf{r}_2(s) = \langle 2, 0, 3\rangle + s\langle 1, 4, 4\rangle$ intersect, and if so, find the point of intersection.

2. (i) (5 pts) Find the vector and scalar projections of **a** onto **b**, and determine whether the angle between **a** and **b** is acute or obtuse.

$$\mathbf{a} = \langle -1, 2, 0 \rangle, \quad \mathbf{b} = \langle 2, 0, 1 \rangle$$

(ii) (3 pts) Use the scalar triple product to determine whether the points A, B, C and D lie in the same plane.

$$A(1, 3, 2), \;\; B(3, -1, 6), \;\; C(5, 2, 0), \quad D(3, 6, -4)$$

3. (i) (6 pts) Compute the angle between the plane through the points $(1, 0, 0), (0, 1, 0)$, and $(0, 0, 1)$ and the yz-plane.

(ii) (3 pts) Find the intersection of the line and the plane.

$$x + y + z = 14, \;\; \mathbf{r}(t) = \langle 1, 1, 0 \rangle + t\langle 0, 2, 4 \rangle$$

4. (i) (5 pts) Parameterize the curve which is the intersection of the plane $y = \dfrac{1}{2}$ with the sphere $x^2+y^2+z^2 = 1$.

(ii) (6 pts) Find the location and velocity at $t = 4$ of a particle whose path satisfies

$$\frac{d\mathbf{r}}{dt} = \left\langle 2t^{-1/2}, 6, 8t \right\rangle, \quad \mathbf{r}(1) = \langle 4, 9, 2 \rangle$$

5. (7 pts) Find an arc length parametrization of the circle of radius 2 with center $(1, 2, 5)$ in a plane parallel to the yz-plane.

   Hint: First parameterize the circle with respect to t, and then use the Fundamental Theorem of Calculus to find t in terms of s.

6. Let C be the curve whose parameterization is given by

$$\mathbf{r}(t) = \langle \cos t, \sin t, \ln \cos t \rangle$$

(i) (3 pts) Find the curvature at the point $(1, 0, 0)$.

Hint: $\kappa(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$

(ii) (5 pts) Find equations of the normal plane and osculating plane of the curve at the point $(1, 0, 0)$.

Hint: $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}, \quad \mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}, \quad \mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Extra Credit Problem (2 points) A plane flying due east at 200 km/h encounters a 40-km/h wind blowing in the north-east direction. The resultant velocity of the plane is the vector sum $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$, where $\mathbf{v}_1$ is the velocity vector of the plane and $\mathbf{v}_2$ is the velocity vector of the wind. Than angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ is $\frac{\pi}{4}$. Determine the resultant *speed* of the plane (the length of the vector $\mathbf{v}$).

Hint: Here I am asking you to find the magnitude of $\mathbf{v}$. Consider the location of the plane in the figure as the origin and write $\mathbf{v}_1$ and $\mathbf{v}_2$ in component form.