1. Consider the set of all multiples of a fixed positive integer n, denoted by

$$n\mathbb{Z} = \{x \in \mathbb{Z} \,|\, x = nk \text{ for some } k \in \mathbb{Z}\}.$$

   Show that $n\mathbb{Z}$ is a subgroup of $\mathbb{Z}$.

2. The set $\{1, 2, \ldots, n-1\}$ is a group under multiplication modulo n if and only if n is prime.

3. (i) Is the set of all 2×2 matrices with real entries a group under matrix multiplication?

   (ii) Is the set of all 2×2 matrices with real entries a group under componentwise addition?

   (iii) Is the set of integers a group under subtraction?

4. Show that if G is a finite group with even number of elements, then there must exist an element $a \in G$ with $a \neq e$ such that $a^2 = e$.

5. Let G be a nonabelian group. Then there exists a pair of elements, say a and b, that do not commute, i.e. $ab \neq ba$.

   (i) Is $ab \in \{e, a, b\}$?

   (ii) Is $ba \in \{e, a, b\}$?

6. The set of all $n \times n$ matrices over $\mathbb{R}$ with determinant equal to 1 is called the **special linear group over** $\mathbb{R}$, and is denoted by $\mathrm{SL}_n(\mathbb{R})$. Show that $\mathrm{SL}_n(\mathbb{R})$ is a group under matrix multiplication. In fact, $\mathrm{SL}_n(\mathbb{R})$ is a subgroup of $\mathrm{GL}_n(\mathbb{R})$, the **general linear group of degree** n **over the real numbers**.

7. Let G be a group. For $a, b \in G$, prove that $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$ if and only if $ab = ba$.

8. Show that the set

$$A = \{f_{m,b} : \mathbb{R} \mapsto \mathbb{R} \,|\, m \neq 0 \text{ and } f_{m,b}(x) = mx + b\}$$

   of **affine functions** from $\mathbb{R}$ to $\mathbb{R}$ forms a group under composition of functions.

9. In the video posted on Canvas in December, I talked about D_4, the set of symmetries of a square. The group D_4 is called the *dihedral group of order 8.* Describe each symmetry in D_3, the set of symmetries in an equilateral triangle. The group D_3 is called the *dihedral group of order 6.*

   (i) Write out the complete Cayley table for D_3.

   (ii) Is D_3 abelian?

10. The analysis carried out for a square and triangle can be done for a regular pentagon or any regular n-gon ($n \geq 3$). The corresponding group is denoted by D_n and is called the *dihedral group of order* $2n$. For $n \geq 3$, describe the elements of D_n.

    Hint: You will need to consider two cases - n even and n odd.