1. For each binary operation $*$ defined on a set below, determine whether or not $*$ gives a group structure on the set. If it is not a group, say which axioms fail to hold.
   (a) Define $*$ on $\mathbb{Z}$ by $a * b = ab$.
   (b) Define $*$ on $\mathbb{Z}$ by $a * b = \max\{a, b\}$
   (c) Define $*$ on $\mathbb{Z}$ by $a * b = a - b$.
   (d) Define $*$ on $\mathbb{Z}$ by $a * b = |ab|$.
   (e) Define $*$ on $\mathbb{R}^+$ by $a * b = ab$.
   (f) Define $*$ on $\mathbb{Q}$ by $a * b = ab$.

2. Let $S = \mathbb{R} \setminus \{-1\}$. Define $*$ on S by
$$a * b = a + b + ab.$$
   Show that $(S, *)$ is a group.

3. Let $G = \{x \in \mathbb{R} \,|\, x > 0 \,\text{and}\, x \neq 1\}$. Define the operation $*$ on G by $a * b = a^{\ln b}$, for all $a, b \in G$. Prove that G is an abelian group under the operation $*$.

4. Let G be a group. Prove that if $x^2 = e$ for all $x \in G$, then G is abelian.