MATH 351 - 01 **Modern Algebra** **Homework Assignment 9**

Instructions

1. There are seven problems, worth 50 points. You must submit solutions by noon on April 26, 2022.
2. You may refer to lecture notes, the textbook, video lectures and lecture notes posted on Canvas.
3. You may not consult any external resources. This means no internet searches, materials from other classes or books or any notes you have taken in other classes. You may not use Google or any other search engines for any reason. You may not use any shared Google documents.
4. You may not discuss this homework assignment or questions related to the assignment with any person outside class.
5. Show ALL your work for full credit.
6. You must sign the front page of the homework assignment and submit it with your solutions. If you do not submit the signed first page, your solutions will NOT be graded and it will be worth ZERO points.
7. Please use A4 sheets to write your solutions and start each problem on a new sheet of paper. Please make sure your solutions are very clear and legible.
8. Please follow the guidelines given in the syllabus on how to prepare and how to submit your homework assignment.

All work submitted is mine and mine alone.

I have read and followed the instructions above.

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Signature

1. (1 point each) Consider the following permutations in S_7.

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 2 & 5 & 4 & 6 & 1 & 7 \end{pmatrix} \text{ and } \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 2 & 1 & 5 & 7 & 4 & 6 & 3 \end{pmatrix}$$

   Compute the following products.

   (i) $\sigma\tau$

   (ii) $\tau\sigma$

   (iii) $\tau^2\sigma$

   (iv) σ^{-1}

   (v) $\sigma\tau\sigma^{-1}$

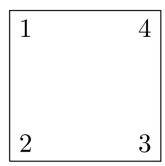
   (vi) $\tau^{-1}\sigma\tau$

2. (6 points) List all elements in the Symmetric Group S_3. Let

$$\alpha = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix} \text{ and } \beta = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}.$$

   Show that $\beta\alpha = \alpha^2\beta$, and then show that S_3 is non-abelian.

3. (10 points) Consider the square region



   Let

$$\rho = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix} \text{ and } \phi = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \end{pmatrix}.$$

   Note that ρ and ϕ denote a 90 degree counterclockwise rotation and a reflection across a horizontal axis, respectively, of the locations of each of the four corners of the square. Show that these two elements generate the entire group D_4 (that is, every element is some combination of the ρ's and ϕ's).

4. (8 points) Write each of the permutations $\sigma\tau$, $\tau\sigma$, $\tau^2\sigma$, σ^{-1}, $\sigma\tau\sigma^{-1}$, and $\tau^{-1}\sigma\tau$ in Problem 1 as a product of disjoint cycles. Write σ and τ as products of transpositions.

5. (2 points each) Find the order of each of the following permutations.

   (i) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 4 & 5 & 3 & 2 & 1 \end{pmatrix}$

   (ii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 6 & 7 & 5 & 1 & 8 & 2 & 3 \end{pmatrix}$

   (iii) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 9 & 8 & 7 & 3 & 4 & 6 & 1 & 2 \end{pmatrix}$

   (iv) $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 4 & 9 & 6 & 5 & 2 & 3 & 1 & 7 \end{pmatrix}$

6. (4 points) Let $\sigma, \tau \in S_n$ be permutations such that $\sigma(k) = k$ and $\tau(k) = k$ for some k with $1 \leq k \leq n$. Show that $\sigma^{-1}(k) = k$ and that $\rho(k) = k$, where $\rho = \sigma\tau$.

7. (8 points) Find the number of cycles of each possible length in S_5. Then find all possible orders of elements in S_5. (Try to do this without having to write out all 120 possible permutations).