Instructions

1. There are seven problems, worth 50 points. You must submit solutions by 12:05pm on April 4, 2022.

2. You may refer to lecture notes, the textbook, video lectures and lecture notes posted on Canvas.

3. You may not consult any external resources. This means no internet searches, materials from other classes or books or any notes you have taken in other classes. You may not use Google or any other search engines for any reason. You may not use any shared Google documents.

4. You may not discuss this homework assignment or questions related to the assignment with any person outside class.

5. Show ALL your work for full credit.

6. You must sign the front page of the homework assignment and submit it with your solutions. If you do not submit the signed first page, your solutions will NOT be graded and it will be worth ZERO points.

7. Please use A4 sheets to write your solutions and start each problem on a new sheet of paper. Please make sure your solutions are very clear and legible.

8. Please follow the guidelines given in the syllabus on how to prepare and how to submit your homework assignment.

All work submitted is mine and mine alone.

I have read and followed the instructions above.

_____
           Signature

1. (10 points) Compute the centralizer in $\mathrm{GL}_2(\mathbb{R})$ of the matrix $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

2. (10 points) Compute the center of $\mathrm{GL}_2(\mathbb{R})$.

3. (6 points) Let $G = \mathrm{GL}_3(\mathbb{R})$. Show that $T = \left\{ \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix} \right\}$ is a subgroup of G.

4. (6 points) Let G be a group and let $a \in G$ be an element of order 12. What is the order of a^j for $j = 2, \ldots, 11$?

5. (6 points) Find the order of the cyclic subgroup of $C^\times$ generated by $1 + i$.

6. (6 points) Prove that any cyclic group with more than two elements has at least two different generators.

7. (6 points) Let G be a group with p^k elements, where p is a prime number and $k \geq 1$. Prove that G has a subgroup of order p.