MATH 351 - 01 **Modern Algebra** **Homework Assignment 4**

Instructions

1. There are five problems, worth 50 points. You must submit solutions by 12:05pm on February 28, 2022.
2. You may refer to lecture notes, the textbook, video lectures and lecture notes posted on Canvas.
3. You may not consult any external resources. This means no internet searches, materials from other classes or books or any notes you have taken in other classes. You may not use Google or any other search engines for any reason. You may not use any shared Google documents.
4. You may not discuss this homework assignment or questions related to the assignment with any person outside class.
5. Show ALL your work for full credit.
6. You must sign the front page of the homework assignment and submit it with your solutions. If you do not submit the signed first page, your solutions will NOT be graded and it will be worth ZERO points.
7. Please use A4 sheets to write your solutions and start each problem on a new sheet of paper. Please make sure your solutions are very clear and legible.
8. Please follow the guidelines given in the syllabus on how to prepare and how to submit your homework assignment.

All work submitted is mine and mine alone.

I have read and followed the instructions above.

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Signature

1. (10 points) Let G be a group. Prove that G is abelian if and only if $(ab)^{-1} = a^{-1}b^{-1}$ for all $a, b \in G$.

2. (10 points) Show that a nonabelian group must have at least five distinct elements.

3. (10 points) The set of all invertible $n \times n$ matrices with entries in $\mathbb{R}$ is called the **general linear group of degree n over the real numbers**, and is denoted by $\mathrm{GL}_n(\mathbb{R})$. Prove that the set $\mathrm{GL}_n(\mathbb{R})$ forms a group under matrix multiplication.

4. (i) (5 points) Let G be a group. We have shown that $(ab)^{-1} = b^{-1}a^{-1}$. Find a similar expression for $(abc)^{-1}$.

   (ii) (5 points) Prove that multiplication of 2×2 matrices satisfies the associative law.

5. (i) (5 points) Write out the multiplication table for $\mathbb{Z}_7^\times$. Note that $\mathbb{Z}_7^\times$ means $U(7)$. Find the identity element and inverse of each element in the group $\mathbb{Z}_7^\times$.

   (ii) (5 points) Write out the addition table for $\mathbb{Z}_8$. Is $\mathbb{Z}_8$ a group under multiplication modulo 8? Justify your answer.