MATH 351 - 01 **Modern Algebra** **Homework Assignment 2**

Instructions

1. There are four problems, worth 50 points. You must submit solutions by 12:05pm on February 14, 2022.
2. You may refer to lecture notes, the textbook, video lectures and lecture notes posted on Canvas.
3. You may not consult any external resources. This means no internet searches, materials from other classes or books or any notes you have taken in other classes. You may not use Google or any other search engines for any reason. You may not use any shared Google documents.
4. You may not discuss this homework assignment or questions related to the assignment with any person outside class.
5. Show ALL your work for full credit.
6. You must sign the front page of the homework assignment and submit it with your solutions. If you do not submit the signed first page, your solutions will NOT be graded and it will be worth ZERO points.
7. Please use A4 sheets to write your solutions and start each problem on a new sheet of paper. Please make sure your solutions are very clear and legible.
8. Please follow the guidelines given in the syllabus on how to prepare and how to submit your homework assignment.

All work submitted is mine and mine alone.

I have read and followed the instructions above.

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Signature

1. (a) (5 points) Find a solution in integers to the equation

$$1064x + 856y = \gcd(1064, 856).$$

(b) (5 points) Let m and $n (\neq 0)$ be integers. Prove that the expression

$$\frac{\gcd(m,n)}{n}\binom{n}{m}$$

is an integer.

2. (a) (5 points) Solve the following linear congruence.

$$7x \equiv 12 \pmod{17}$$

(b) (5 points) Show that 19 is a divisor of $(18! + 1)$.

3. (10 points) Find the indicated roots and locate them graphically.
$(-2\sqrt{3} - 2i)^{\frac{1}{4}}$.

4. (5 points each)

(a) Let n be a positive integer. For integers, a, b we define $a \sim b$ if $n \mid (a - b)$. Show that $\sim$ is an equivalence relation on the set of integers.

(b) Let S be the set of all ordered pairs (m, n) of positive ietegers. For $(a_1, a_2) \in S$ and $(b_1, b_2) \in S$, define $(a_1, a_2) \sim (b_1, b_2)$ if $a_1 + b_2 = a_2 + b_1$. Show that $\sim$ is an equivalence relation.

(c) On $\mathbb{R}^2$, define $(a_1, a_2) \sim (b_1, b_2)$ if $a_1^2 + a_2^2 = b_1^2 + b_2^2$. Is $\sim$ an equivalence relation?

(d) Let A be the set of all integers and let B be the set of all nonzero integers. On the set $S = A \times B$ of ordered pairs, define $(m, n) \sim (p, q)$ if $mq = np$. Is $\sim$ an equivalence relation?